Uncertainty Quantification of Power and Temperature for Multiprocessor System Design

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Outline

Sneak Peek

Sparse grids

Probability space

Sergiu Rafiliu

Monaco

Polynomial chaos

Temperature

Leakage

Uncertainty Quantification

Ladies

Multiprocessors

Karhunen-Loève

Intel

Ashton Kutcher

Sergey Smolyak

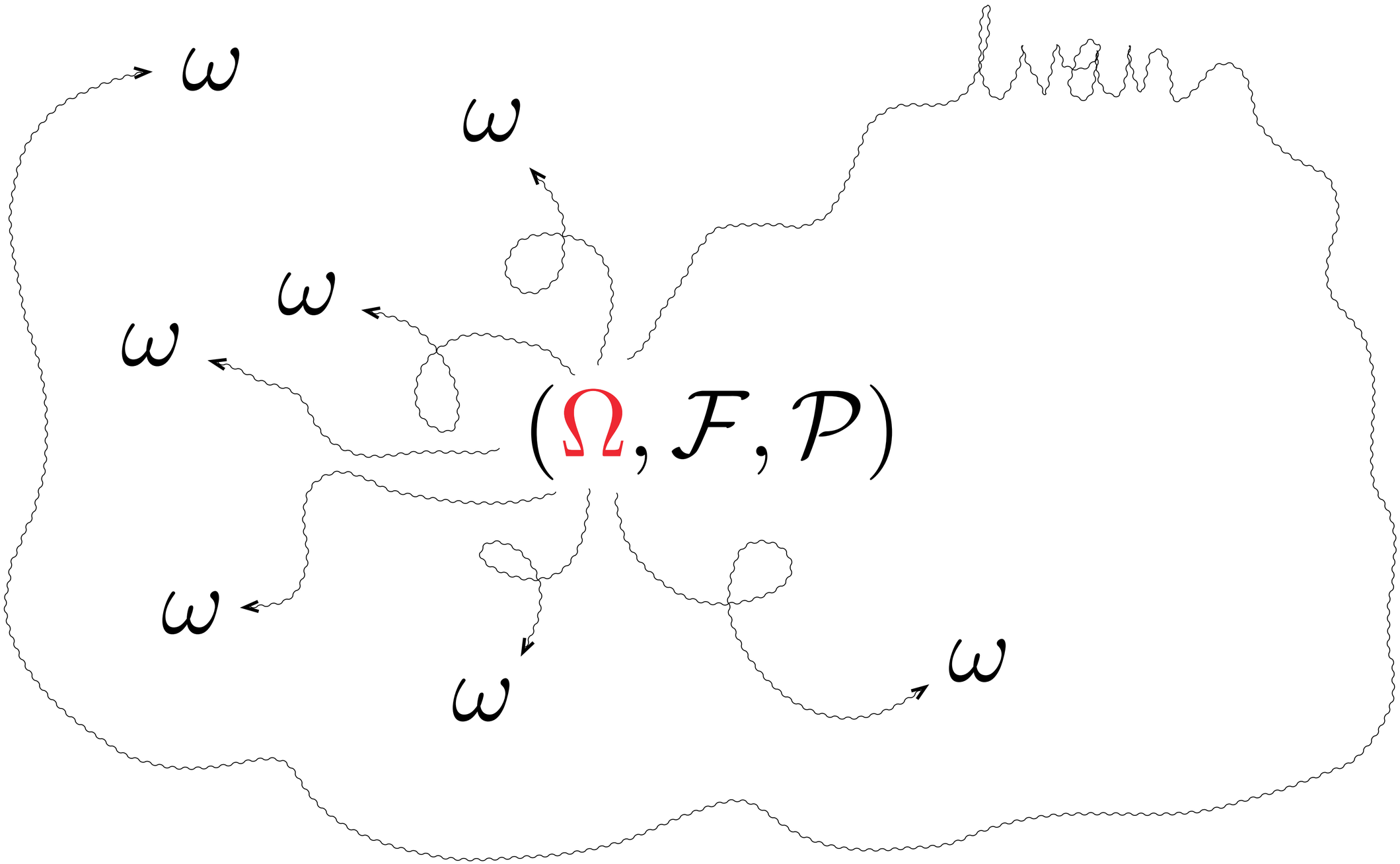
Power

Alpha & omega

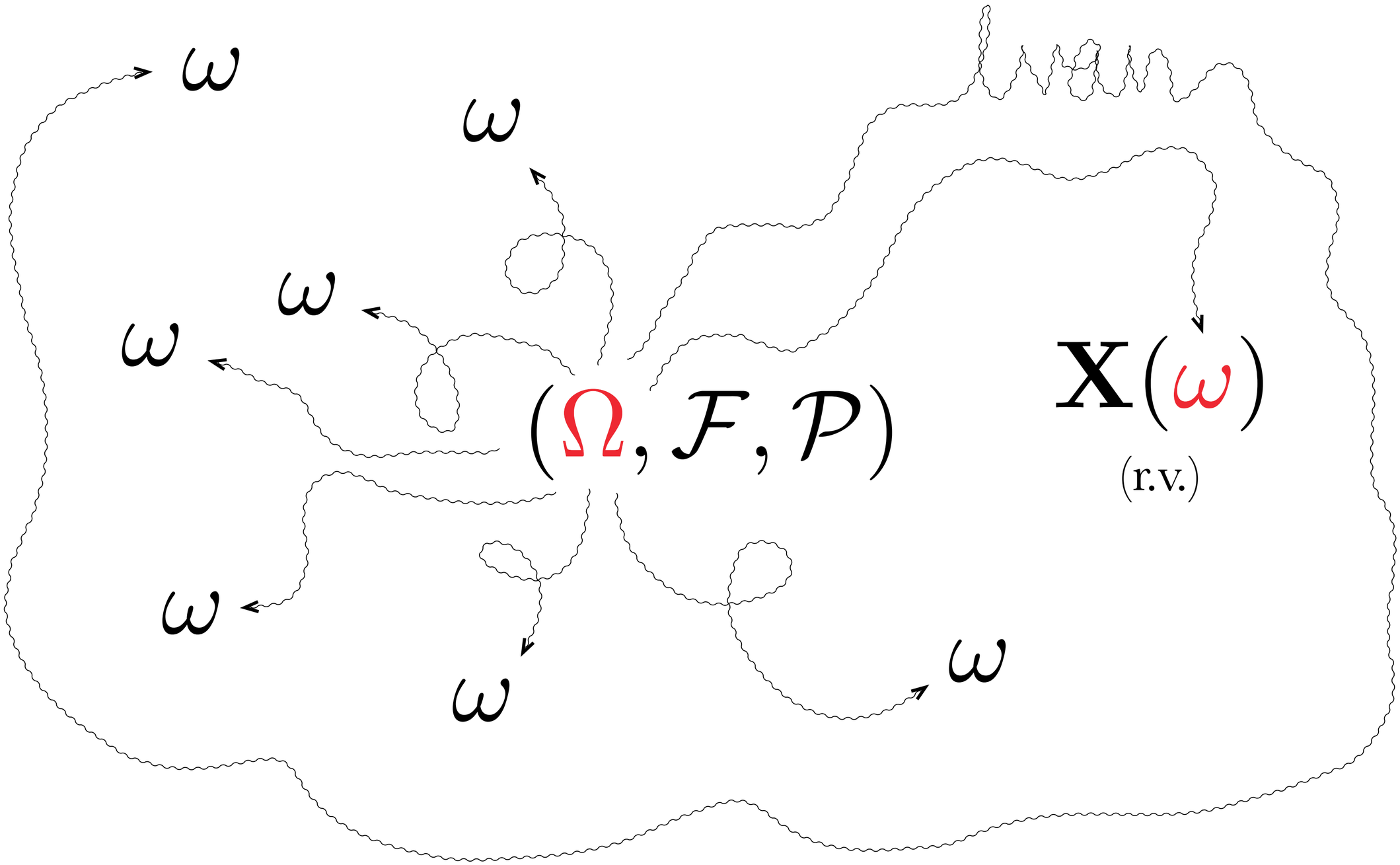
Probability Space

$$(\Omega, \mathcal{F}, \mathcal{P})$$

Probability Space



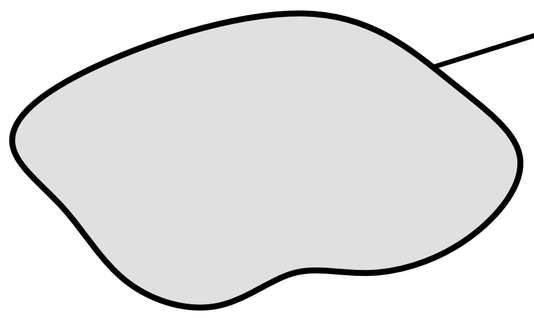
Probability Space



Uncertainty Quantification

UQ

~~Real-Time System~~ [1]

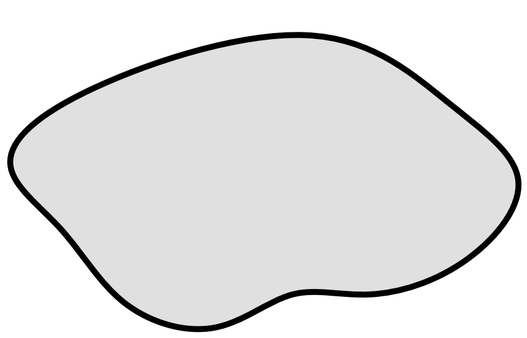


[1] S. Rafiliu et al., ECRTS, 2011.

Uncertainty Quantification

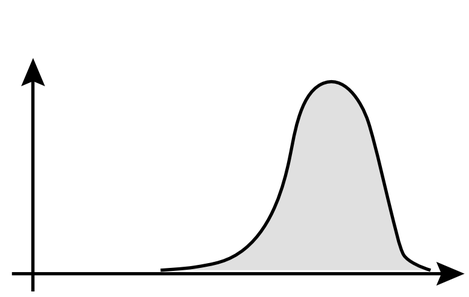
$\mathbf{U}(\omega)$

Uncertain input



Some system

$\mathbf{\Theta}(\omega)$

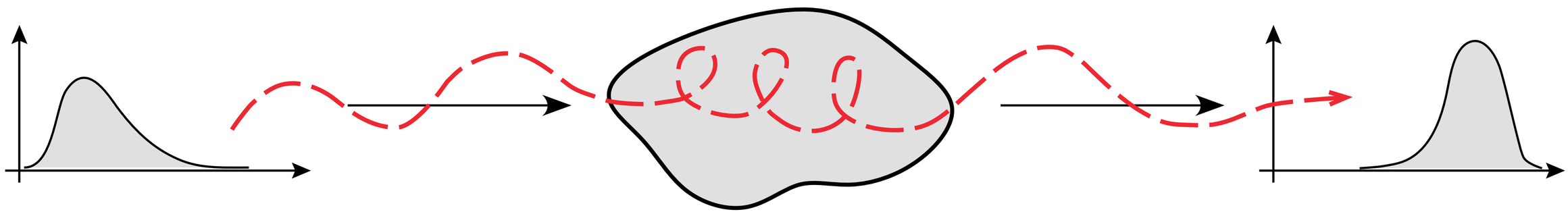


Uncertain output

Uncertainty Quantification

-- UQ

$\mathbf{U}(\omega)$ $\mathbf{\Theta}(\omega)$



Uncertain input

Some system

Uncertain output

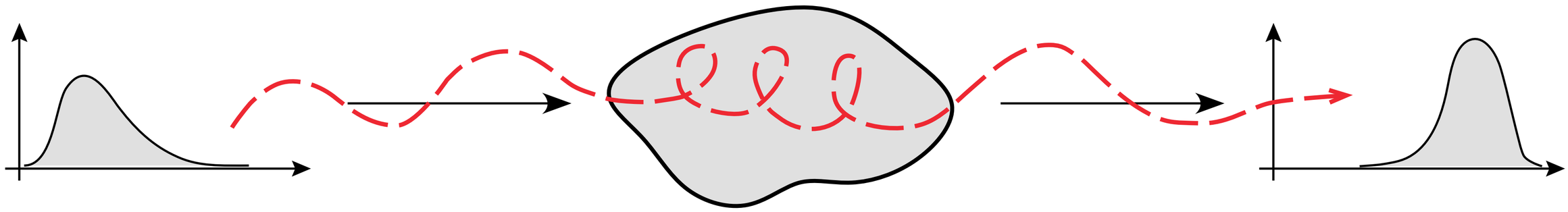
Uncertainty Quantification

-- UQ

$\mathbf{U}(\omega)$

$\mathbf{P}(\omega)$

$\mathbf{\Theta}(\omega)$



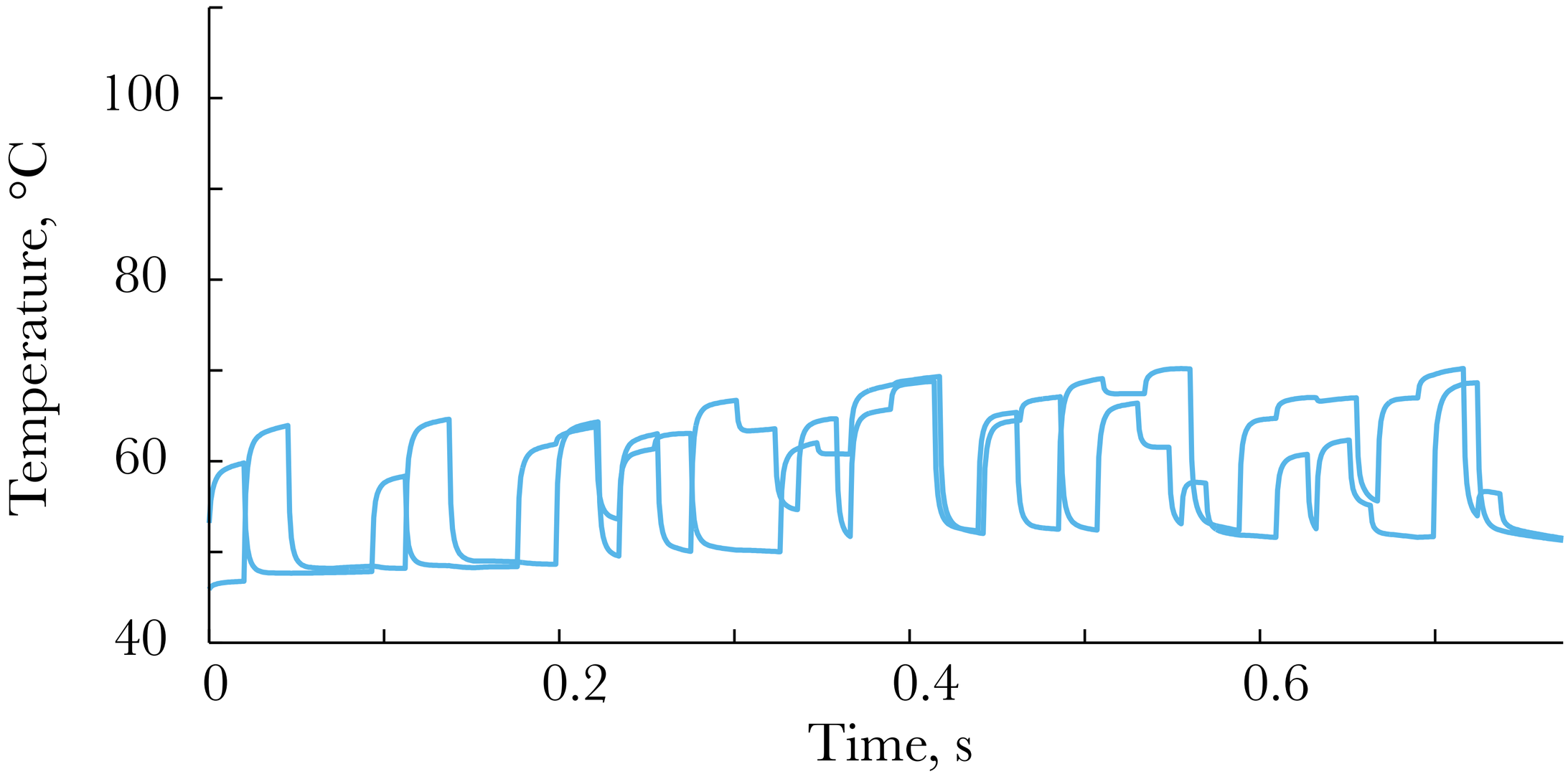
Uncertain input

Multiprocessor system

Power & temperature

Motivation

Consider leakage under process variation.



Motivation

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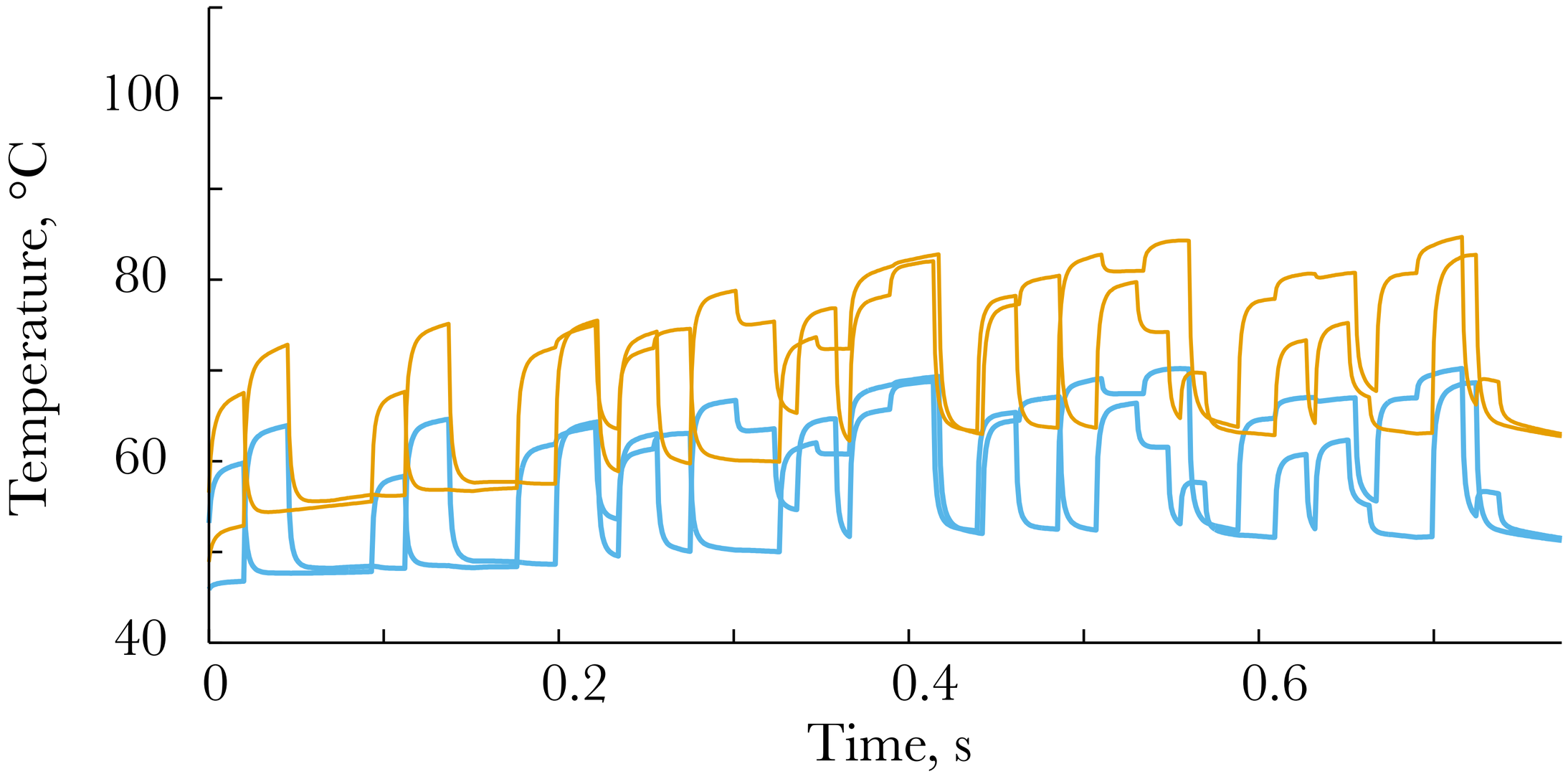

Motivation

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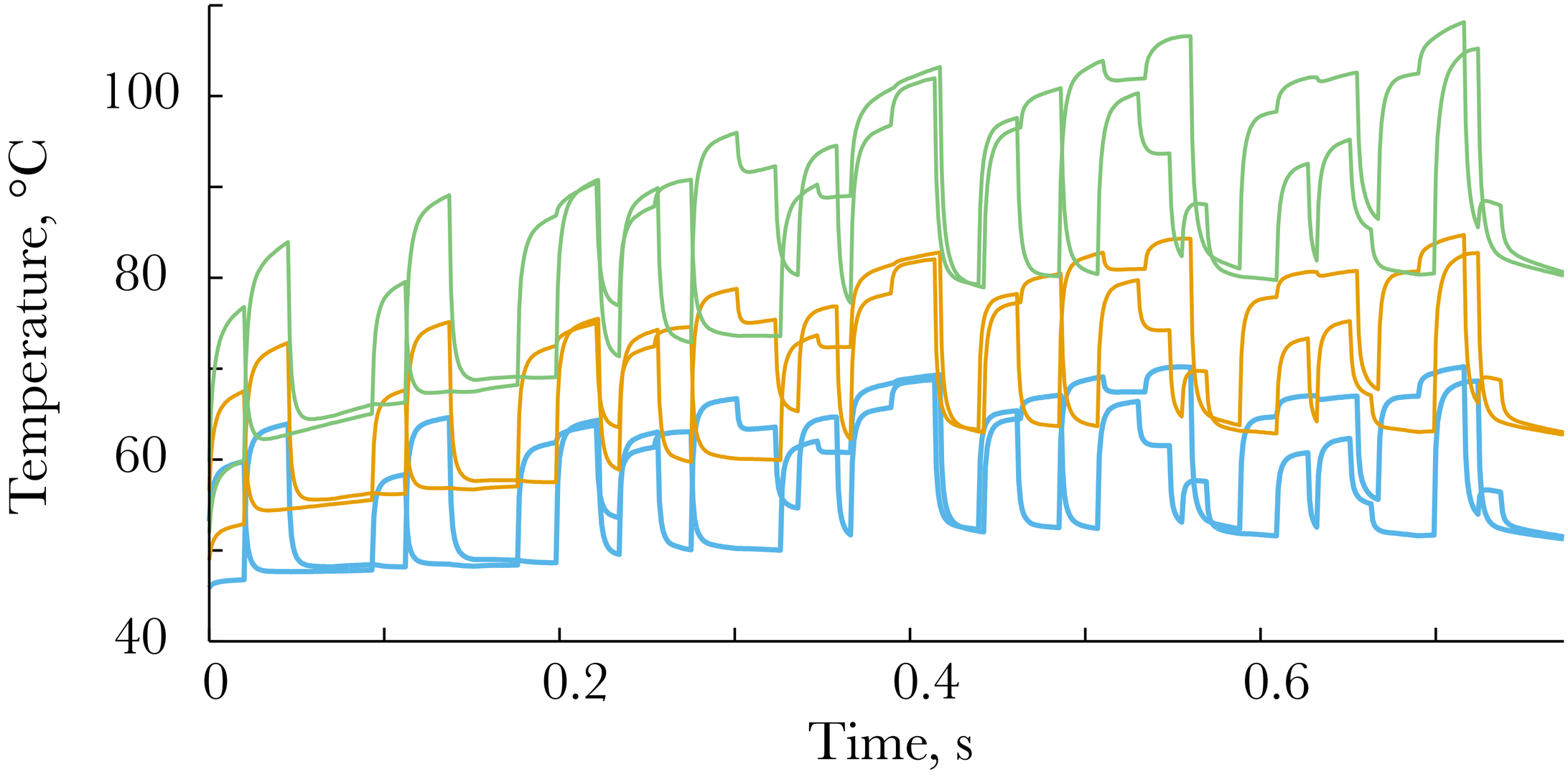

Motivation

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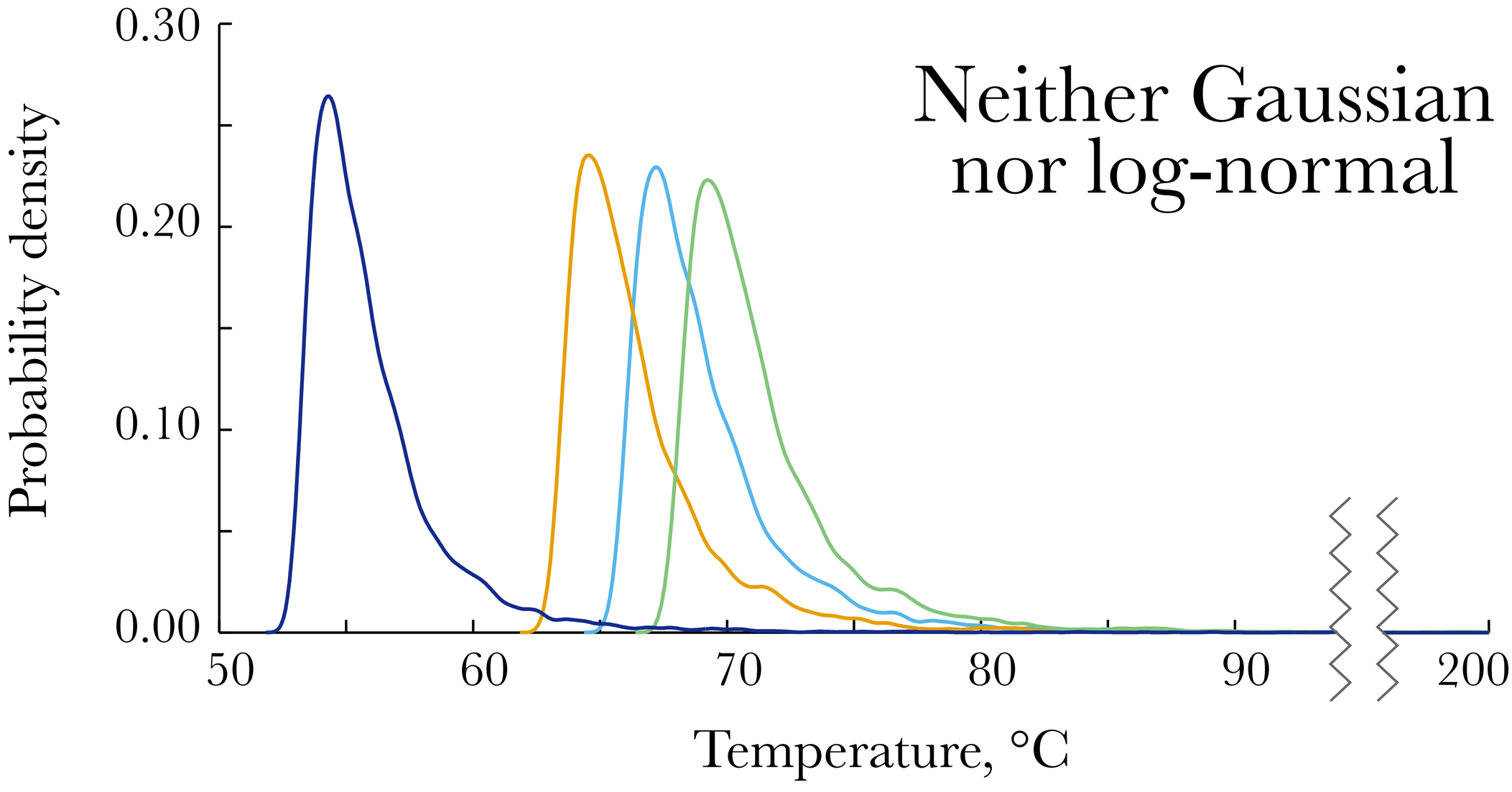


Neither Gaussian nor log-normal

Ultimate Objective

To be certain.

Ultimate Objective

$\mathbf{U}(\omega)$

$\mathbf{P}_{\mathrm{dyn}}$

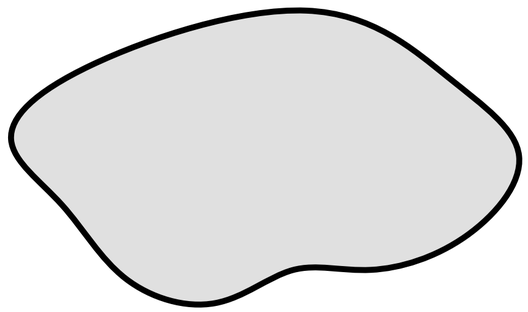
The user

(the designer,
if you will)

Ultimate Objective

$\mathbf{U}(\omega)$
$\mathbf{P}_{\text{dyn}}$

+



The user
(the designer,
if you will)

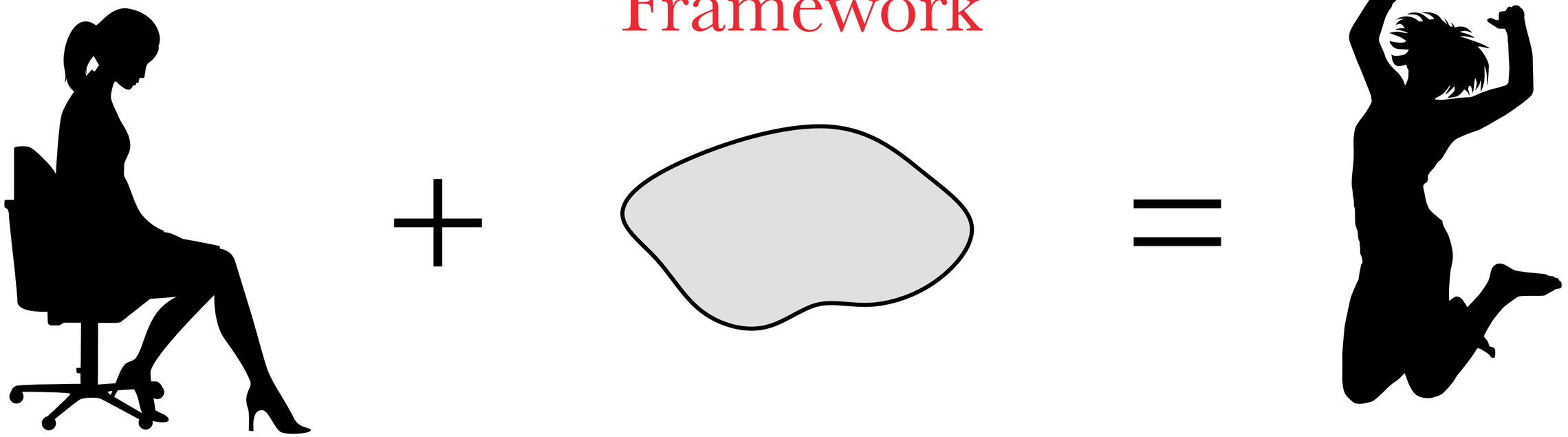
Our UQ
Framework

Ultimate Objective

$\mathbf{U}(\omega)$
$\mathbf{P}_{\text{dyn}}$

$\mathbf{P}(\omega)$
$\mathbf{\Theta}(\omega)$

Our UQ Framework



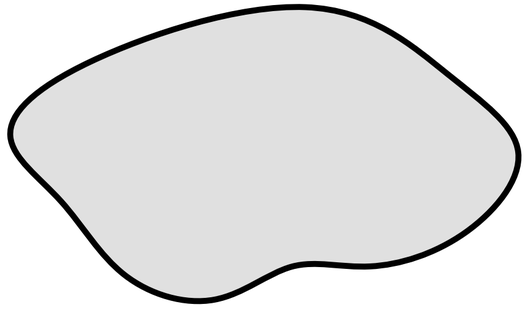
The user
(the designer, if you will)

The same user
(although overexcited)

Monte Carlo

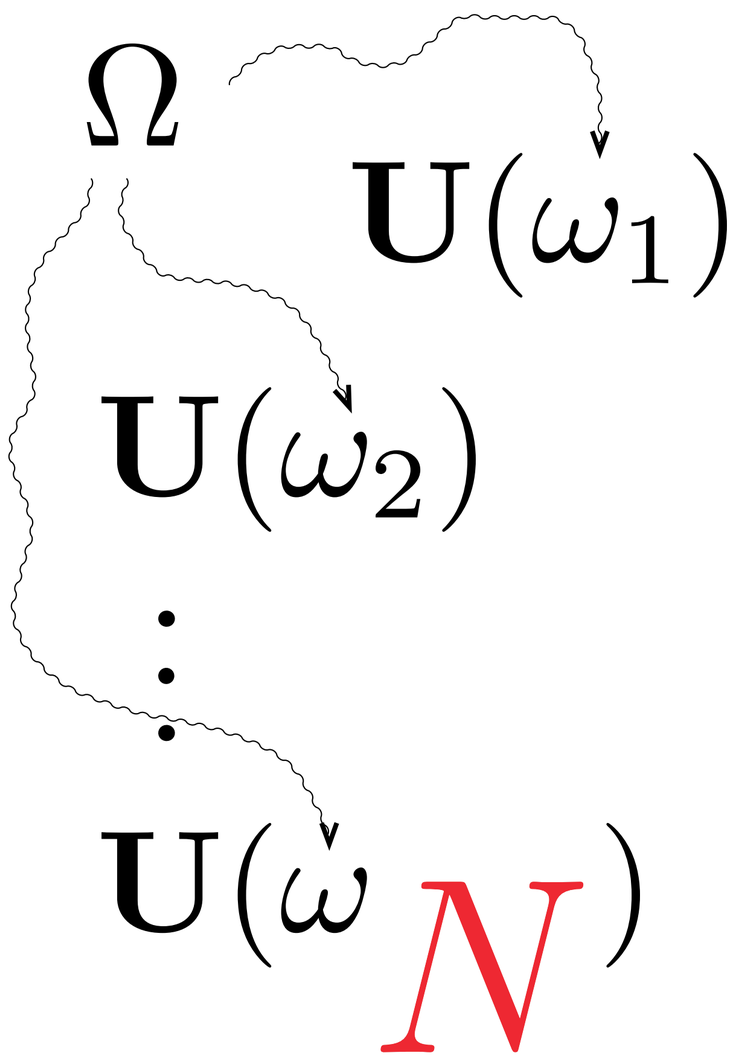
Monte Carlo

Ω



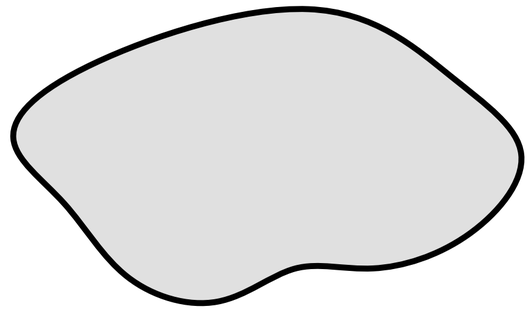
Multiprocessor
system

Monte Carlo



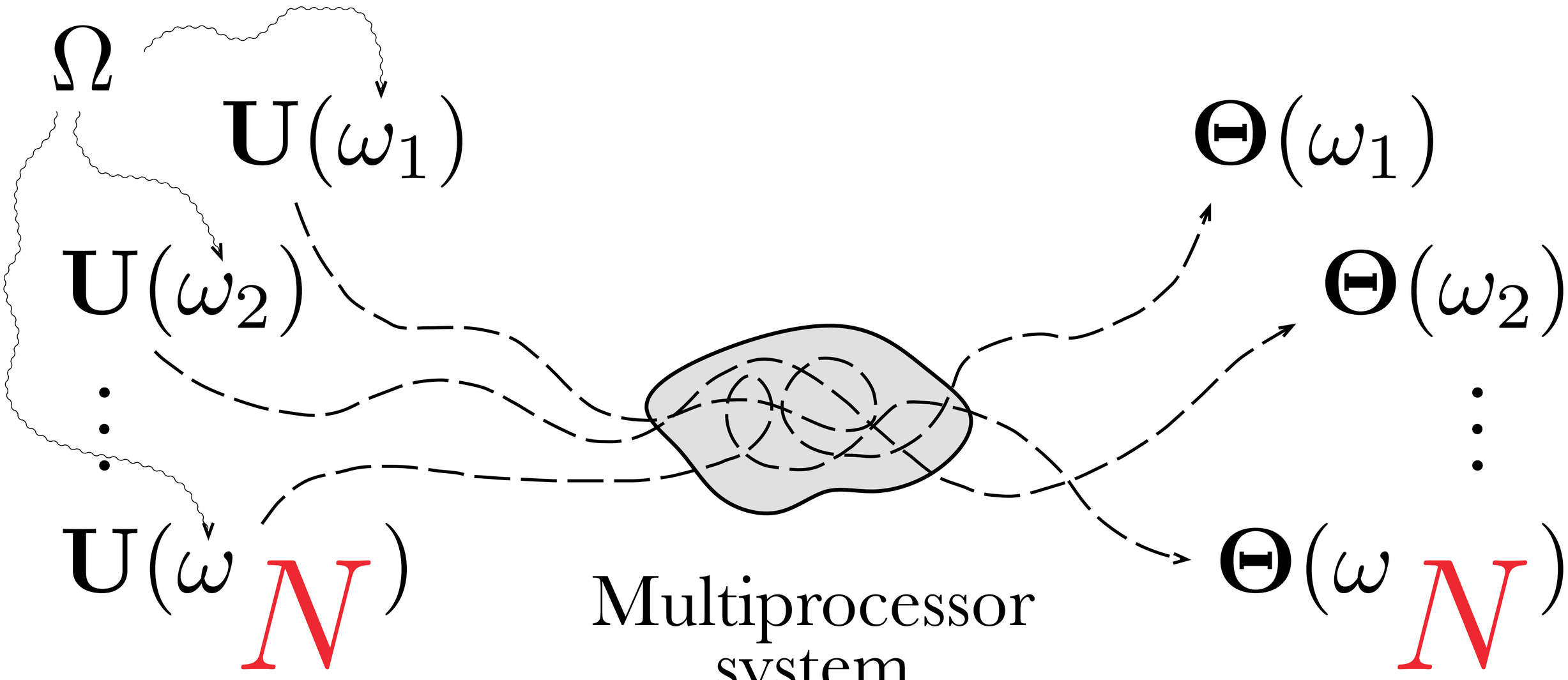
# of samples

(Yes, it's really that huge!)



Multiprocessor system

Monte Carlo



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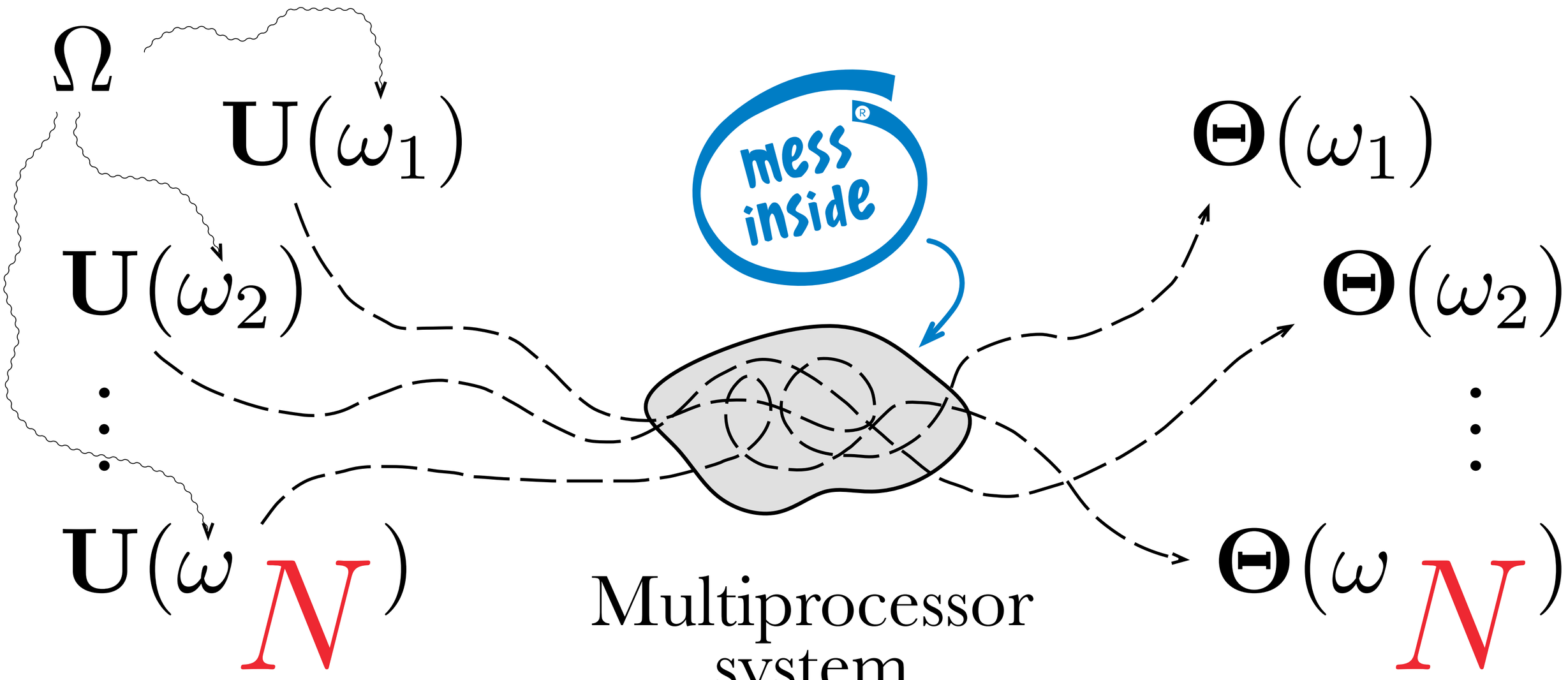
Monte Carlo



# of samples

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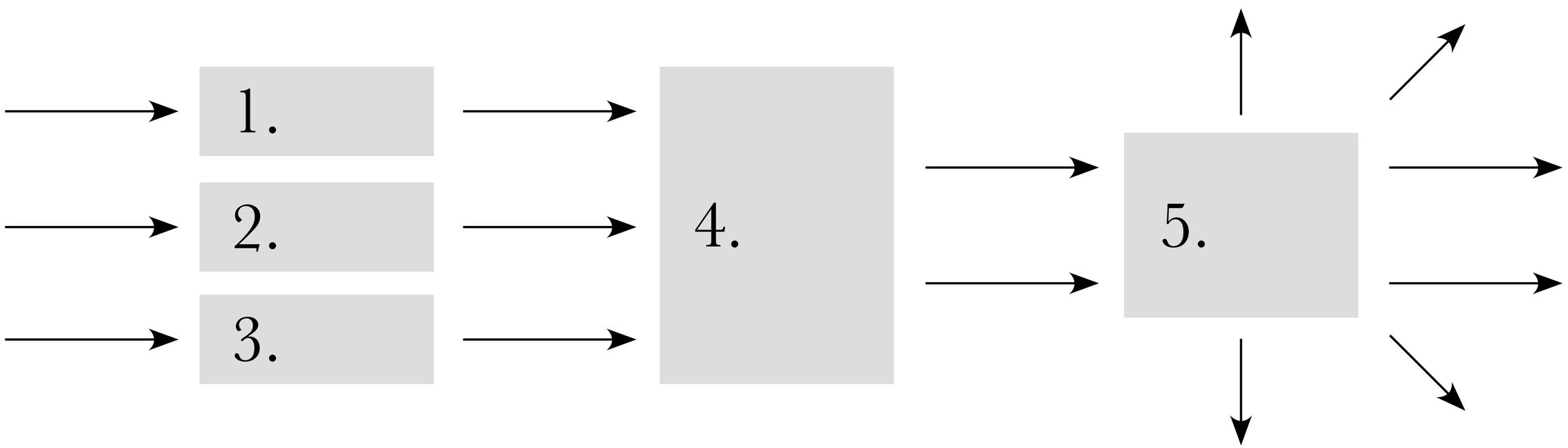
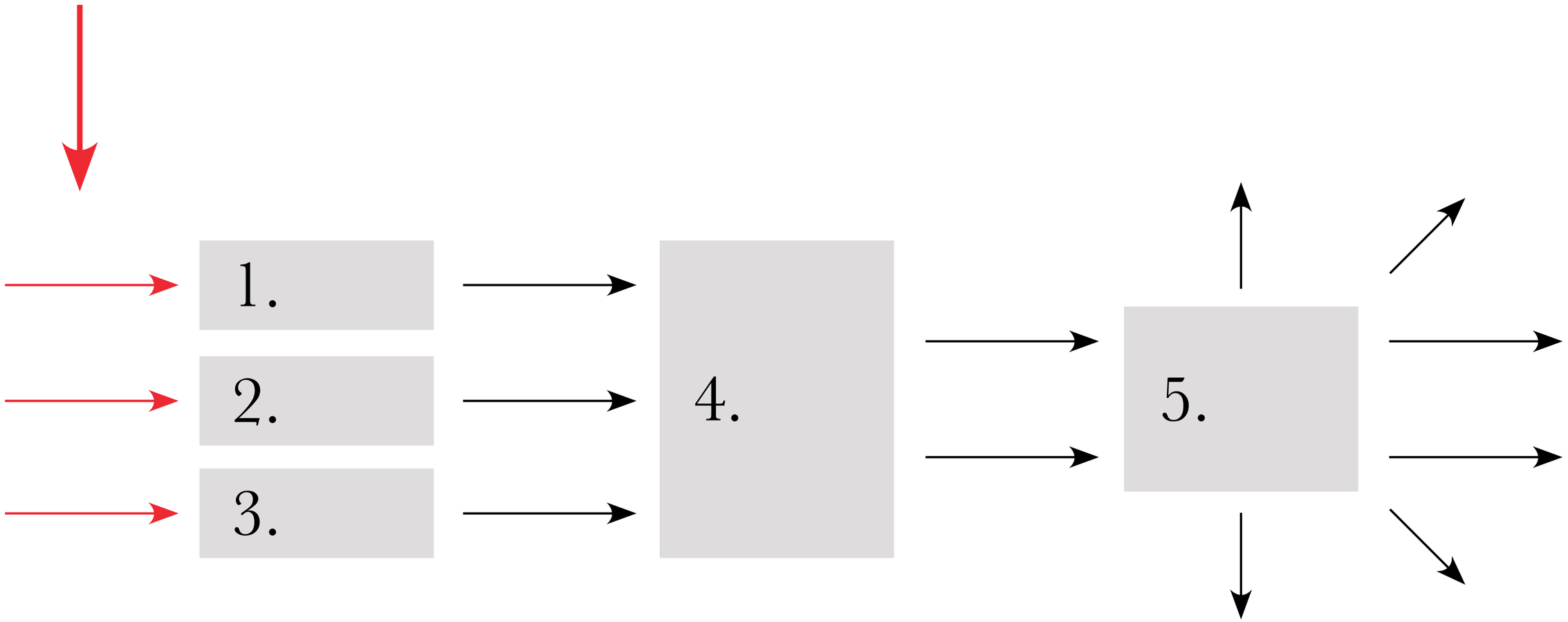
Monte Carlo

Multiprocessor
system

Overview of the Framework



Inputs to the Framework



Inputs to the Framework

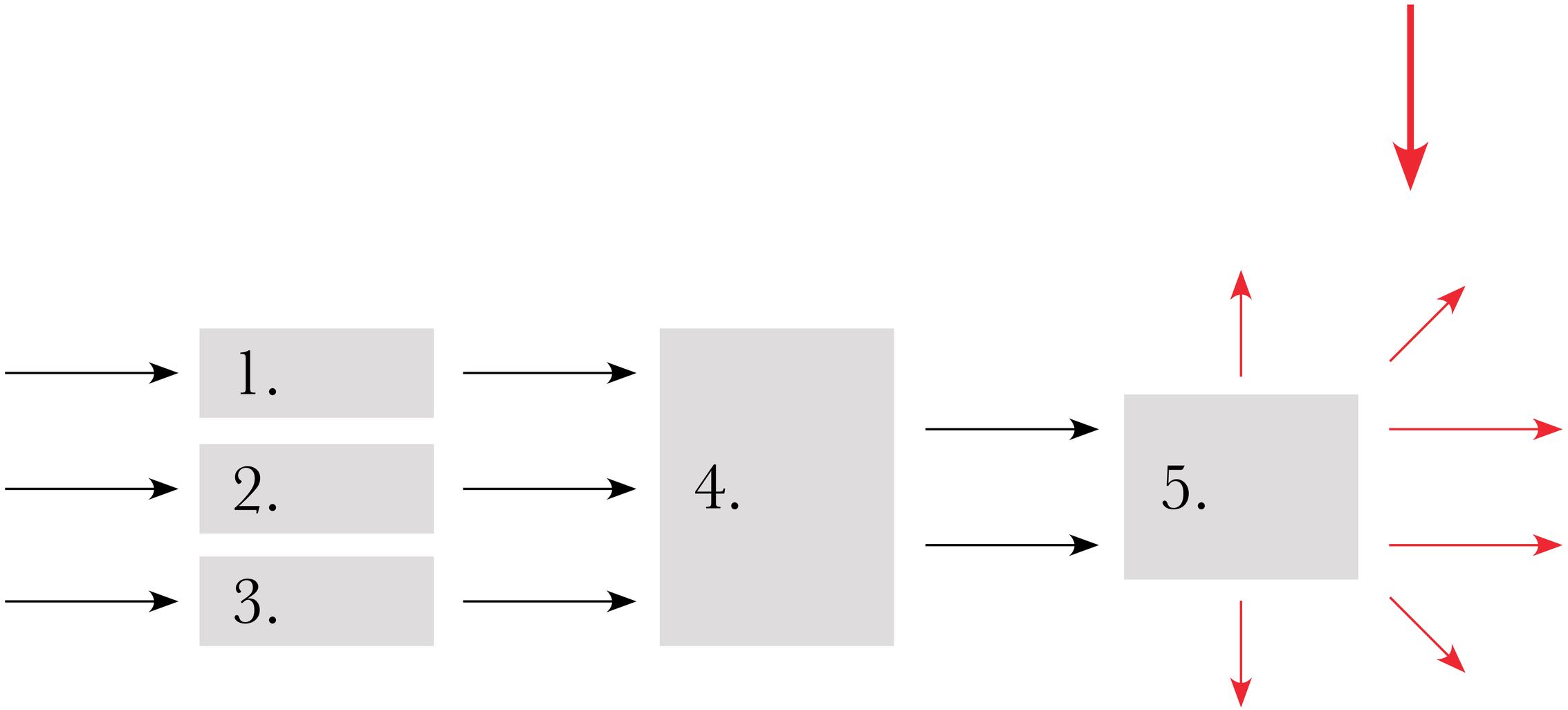
From top to bottom: uncertain parameters, a power model, and a thermal specification.

$\mathbf{U}(\omega)$ ⟶ 1. ——

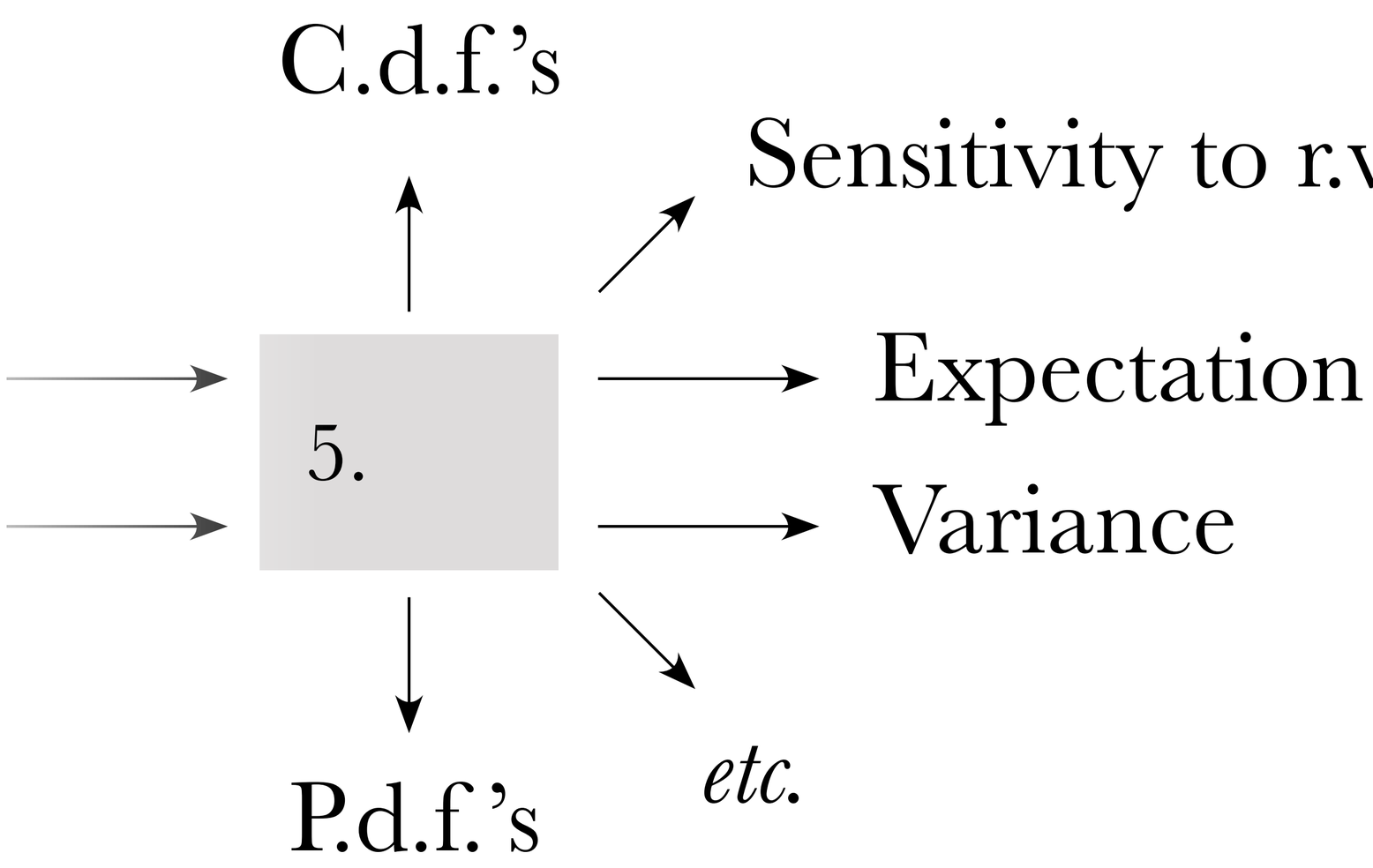
$\mathbf{P}$ ⟶ 2. ——

$\mathcal{S}$ ⟶ 3. ——

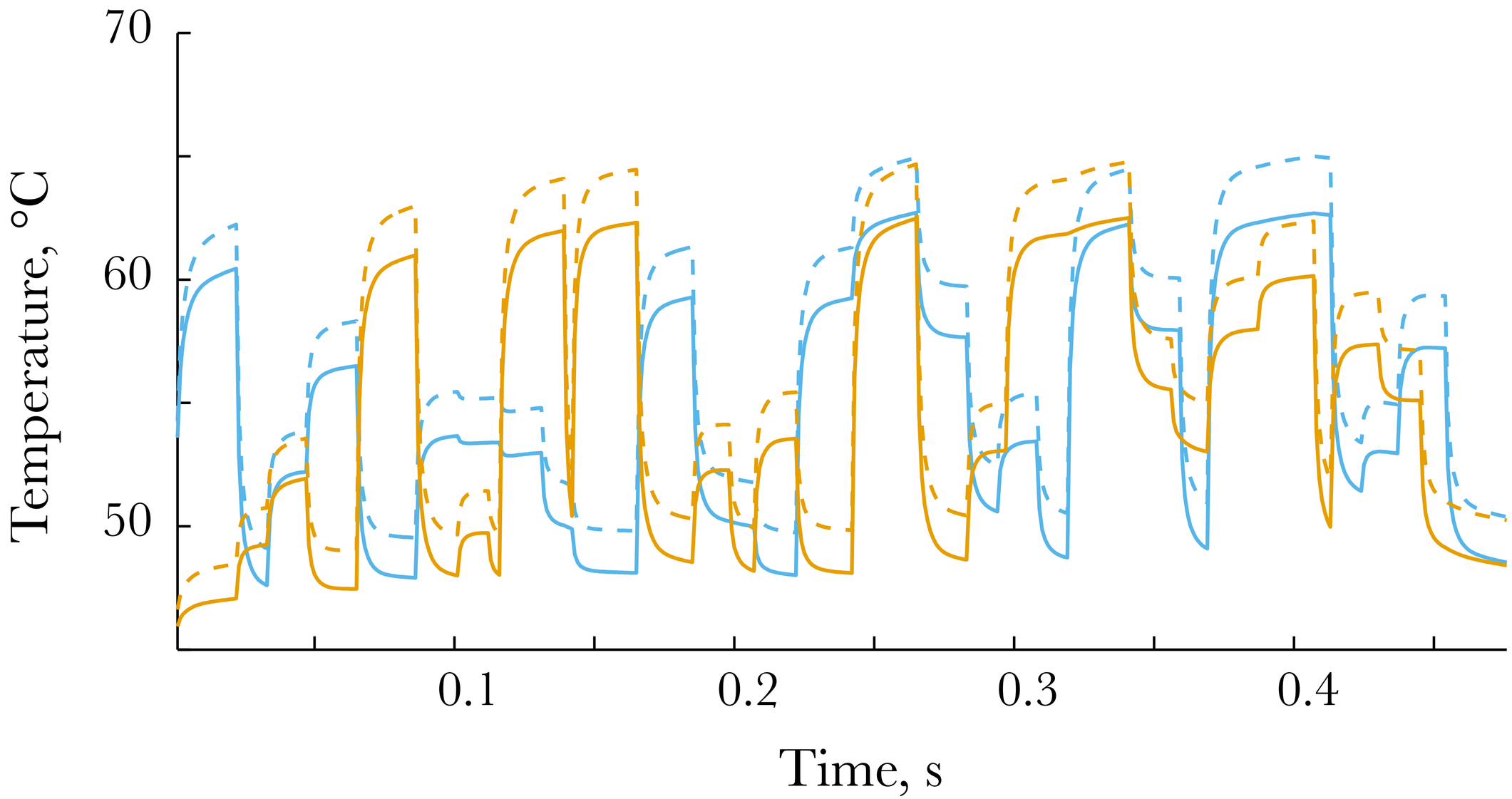
Outputs of the Framework



Outputs of the Framework



Outputs of the Framework



Our Accuracy

Comparing with 10000 MC simulations…

1%	2%	2%
error for expectation.	error for variance.	error for density.

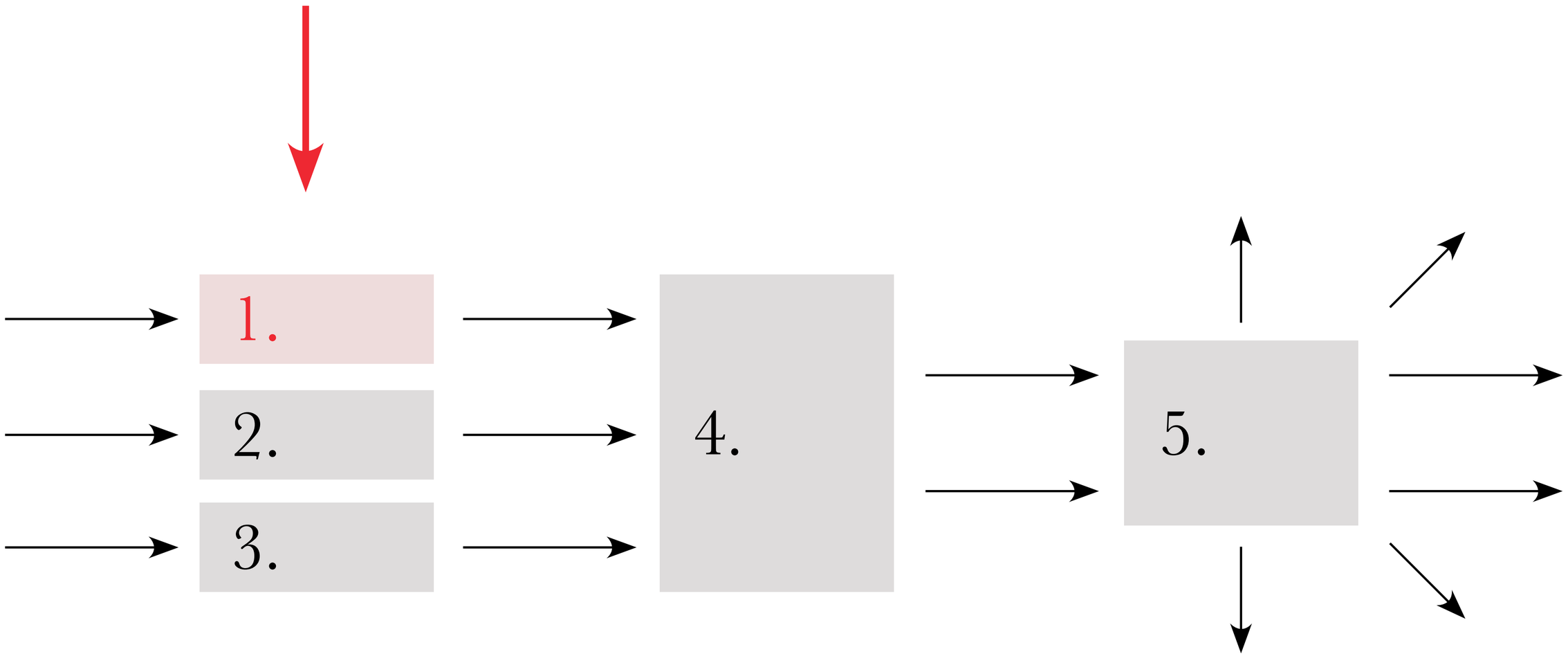
Our Speed

Comparing with 10000 MC simulations…

10^4–$10^5\times$

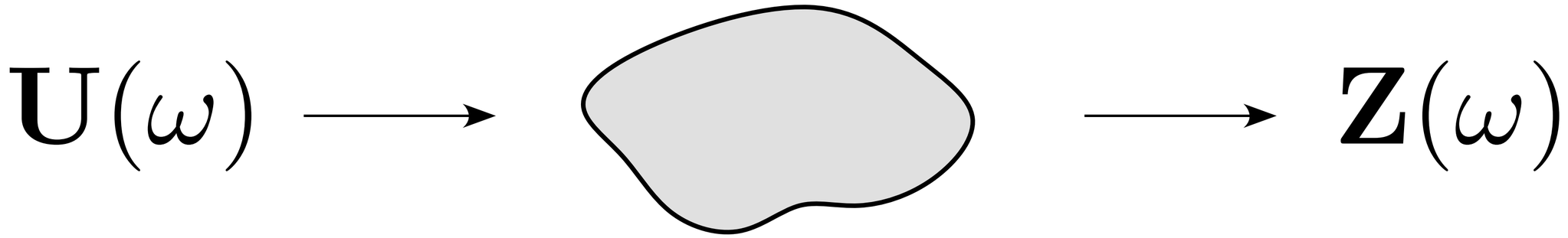
speedup for various
platforms and time spans.

Stage 1: Independence



Stage 1: Independence

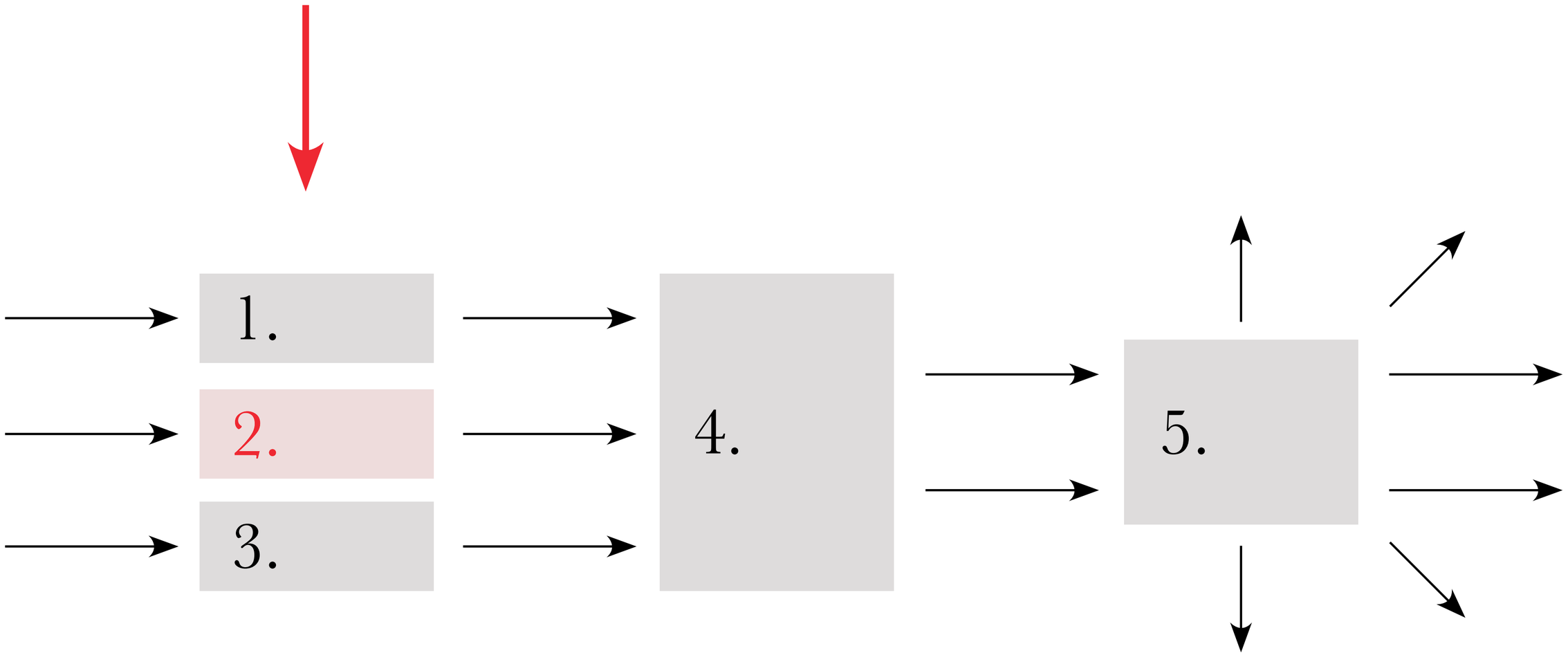
Karhunen-Loève
expansion



Many
correlated

A few
independent

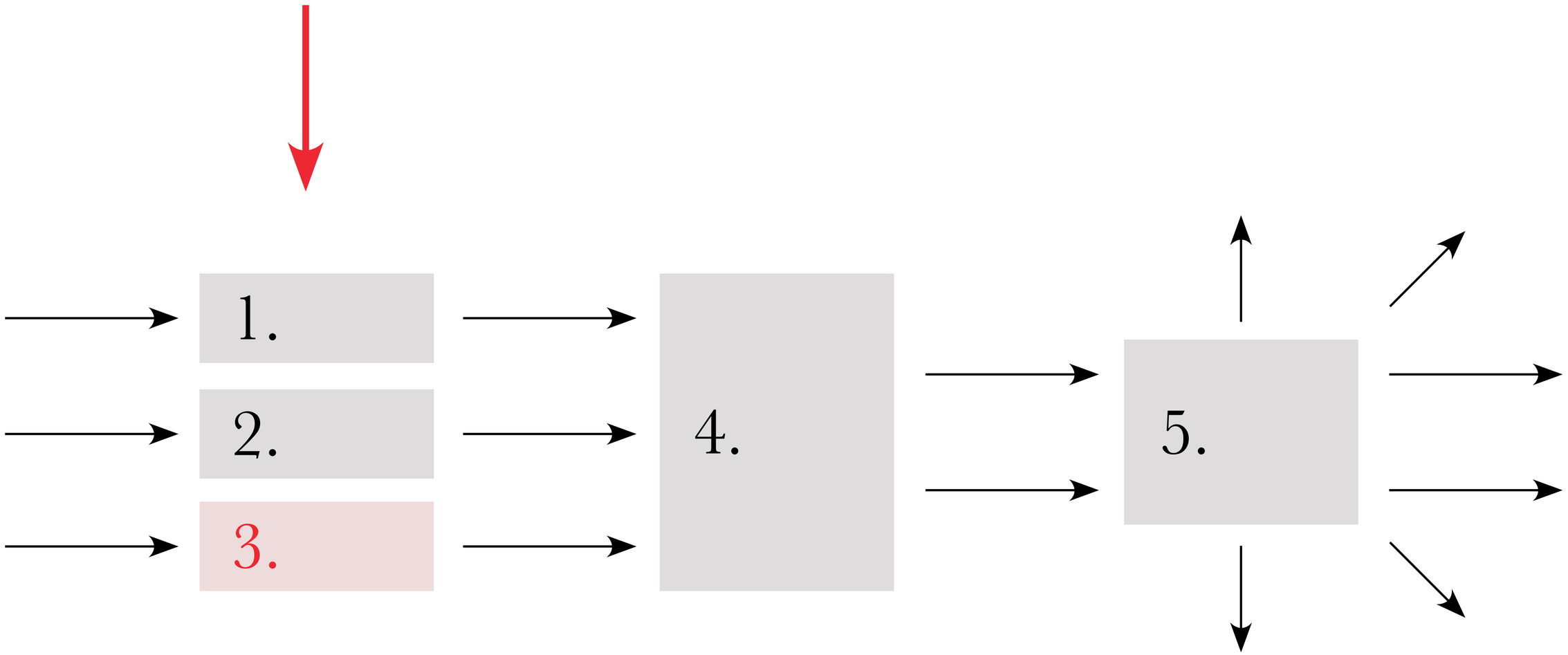
Stage 2: Power Model



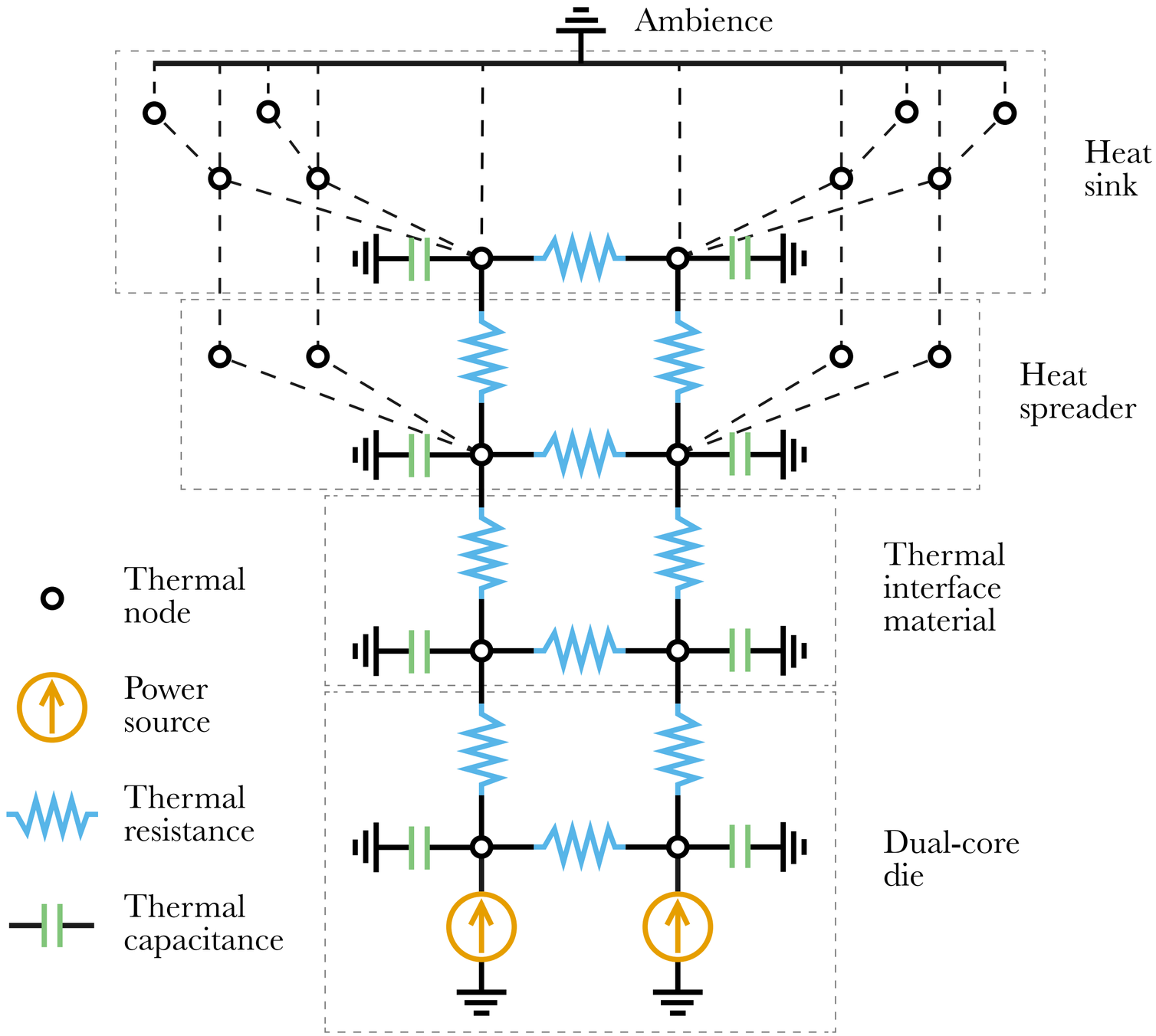
Stage 2: Power Model

$$\mathbf{P}(t, \mathbf{\Theta}(t, \omega), \mathbf{U}(\omega))$$

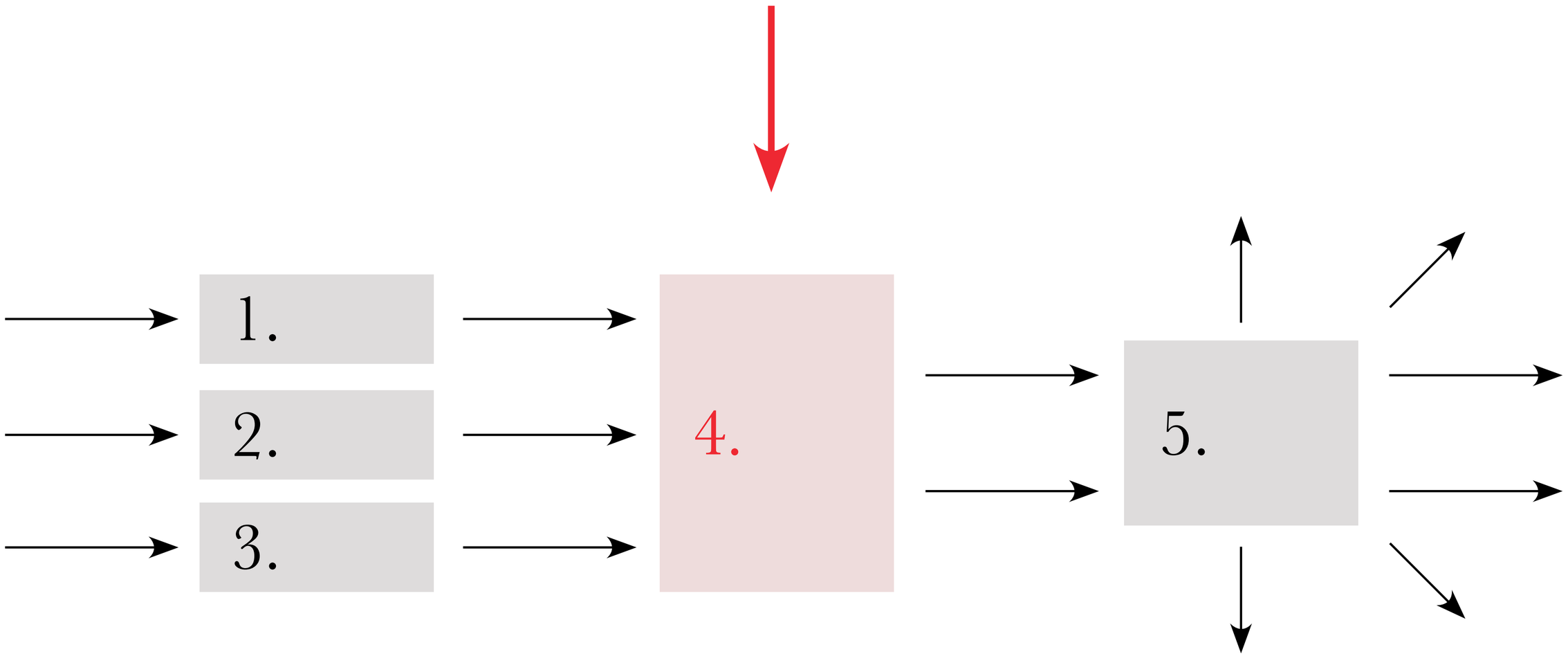
Stage 3: Thermal Model



Stage 3: Thermal Model



Stage 4: Surrogate Model



Polynomial Chaos

"It has been said that something as small as the flutter of a butterfly's wing can ultimately cause a typhoon halfway around the world."

— Chaos Theory

Polynomial Chaos

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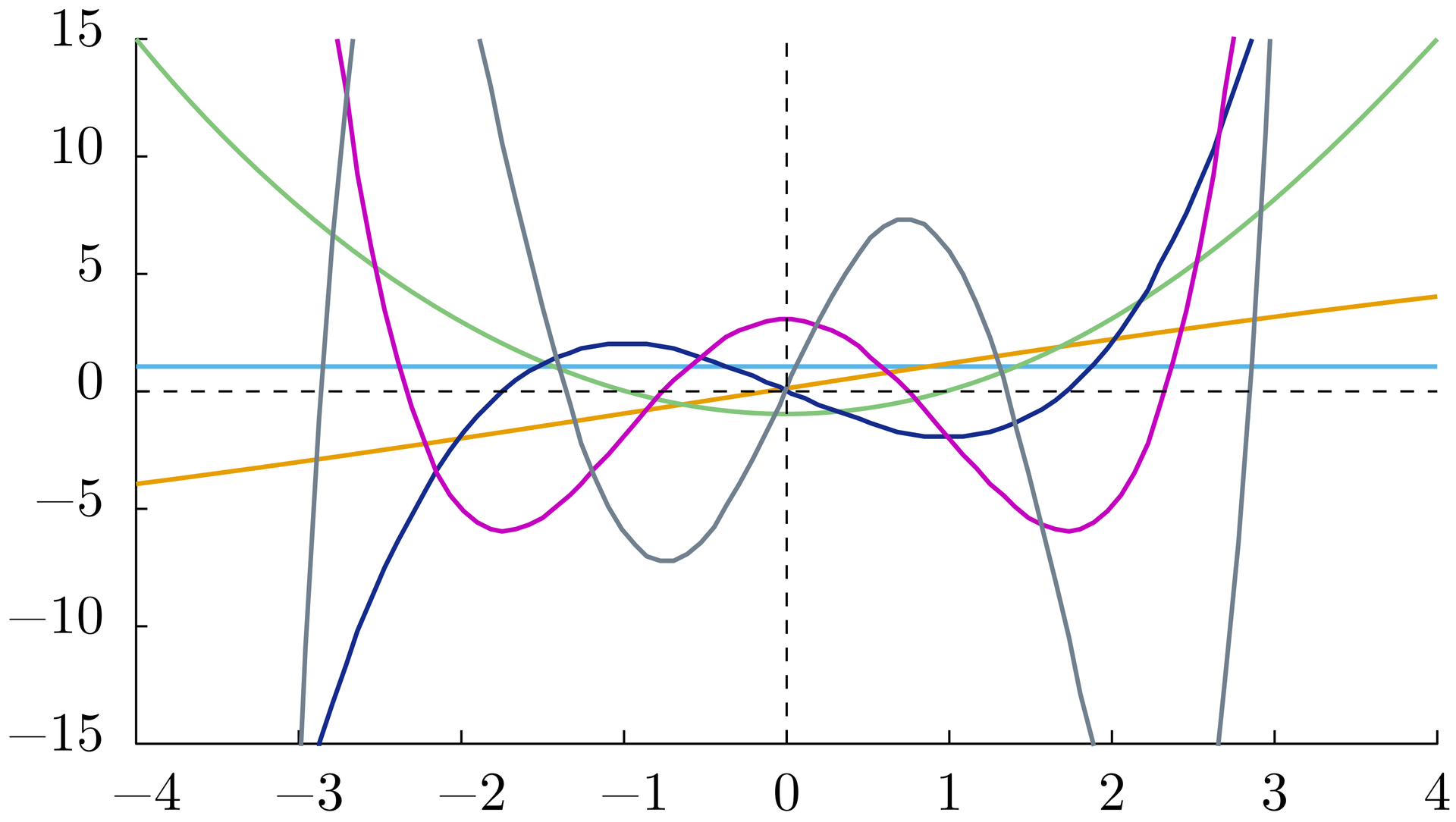
Polynomial Chaos



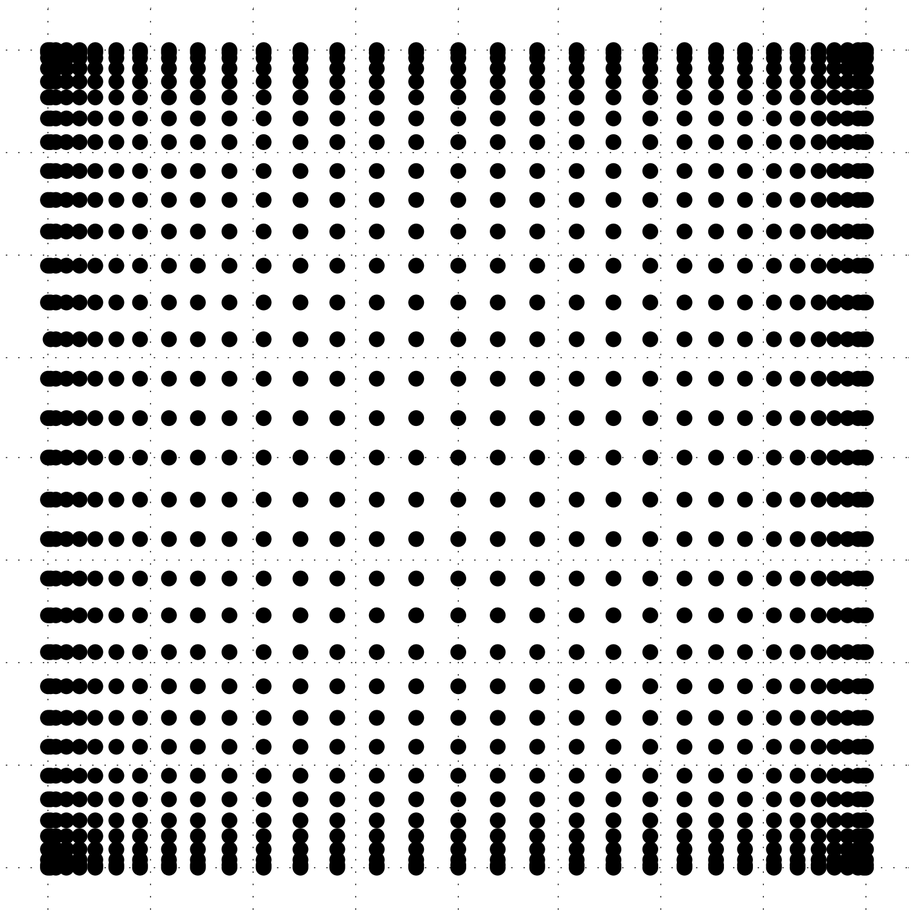
Polynomial Chaos

$$\mathbf{\Theta}(t,\omega) = \sum_{i=1}^{\infty} \boldsymbol{\gamma}_i(t)\ \Phi_i(\mathbf{Z}(\omega))$$

Numerical Integration

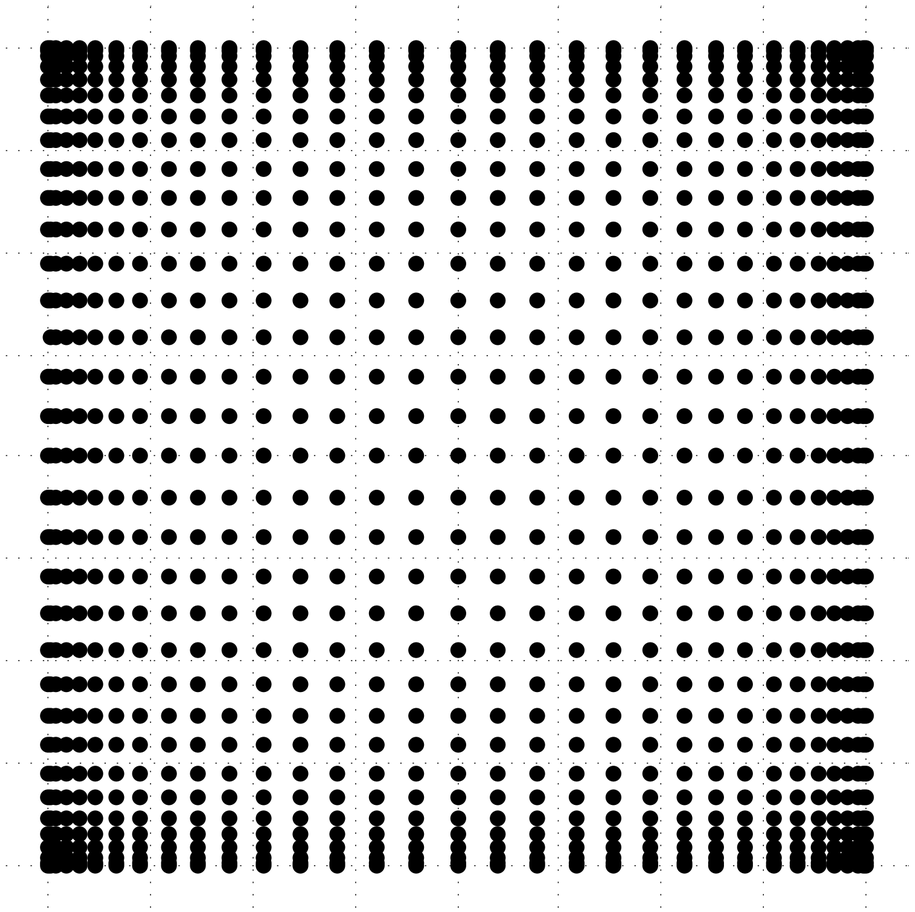
$$\gamma = \int f(u)\, du \approx \sum_i f(\hat{u}_i)\, w_i$$

Multidimensional Integration

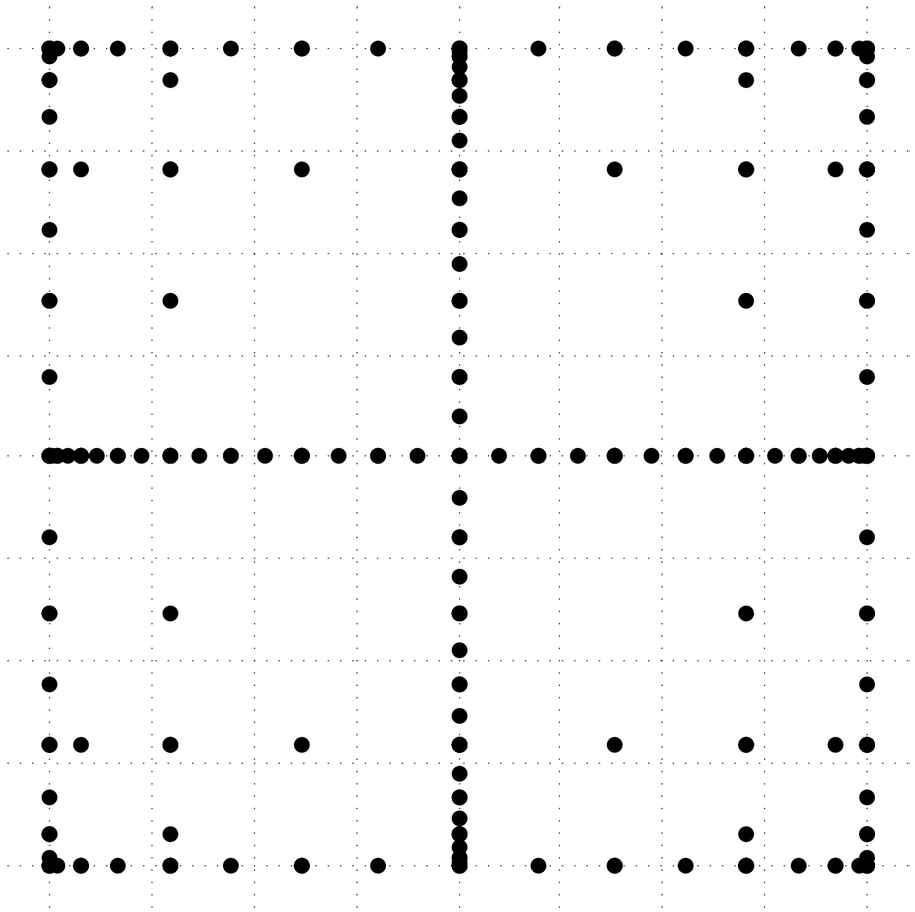


Tensor product

Multidimensional Integration



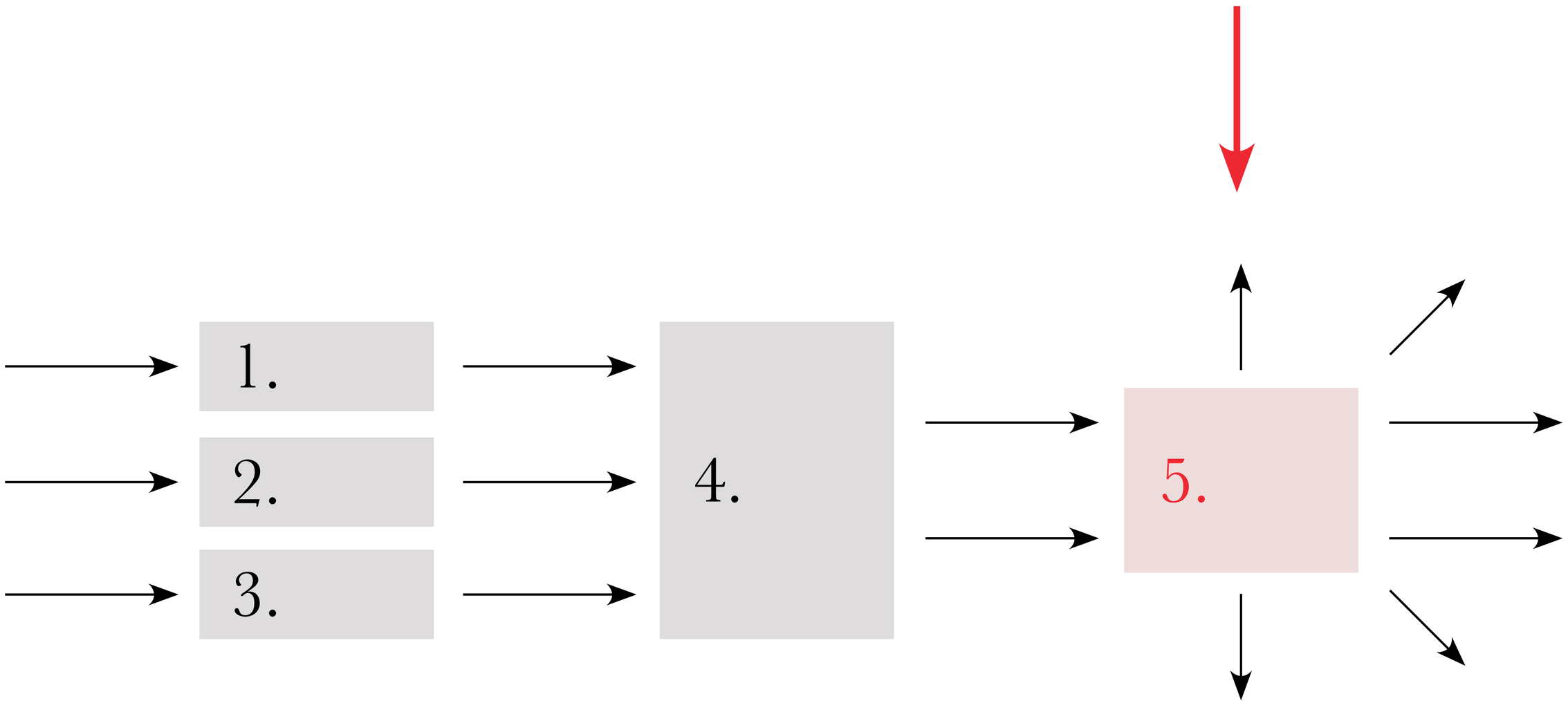
Tensor product



Sparse grid

Sparse Grids = Sergey Smolyak

Stage 5: Post-processing



Stage 5: Post-processing

$$\mathbb{E}[\boldsymbol{\Theta}(t,\omega)] = \boldsymbol{\gamma}_1(t)$$

$$\mathbb{V}\mathrm{ar}[\boldsymbol{\Theta}(t,\omega)] = \sum_{i=2}^{N} \eta_i \boldsymbol{\gamma}_i^2(t)$$

Thank yoUQestions?