Statistical Analysis of Process Variation Based on Indirect Measurements

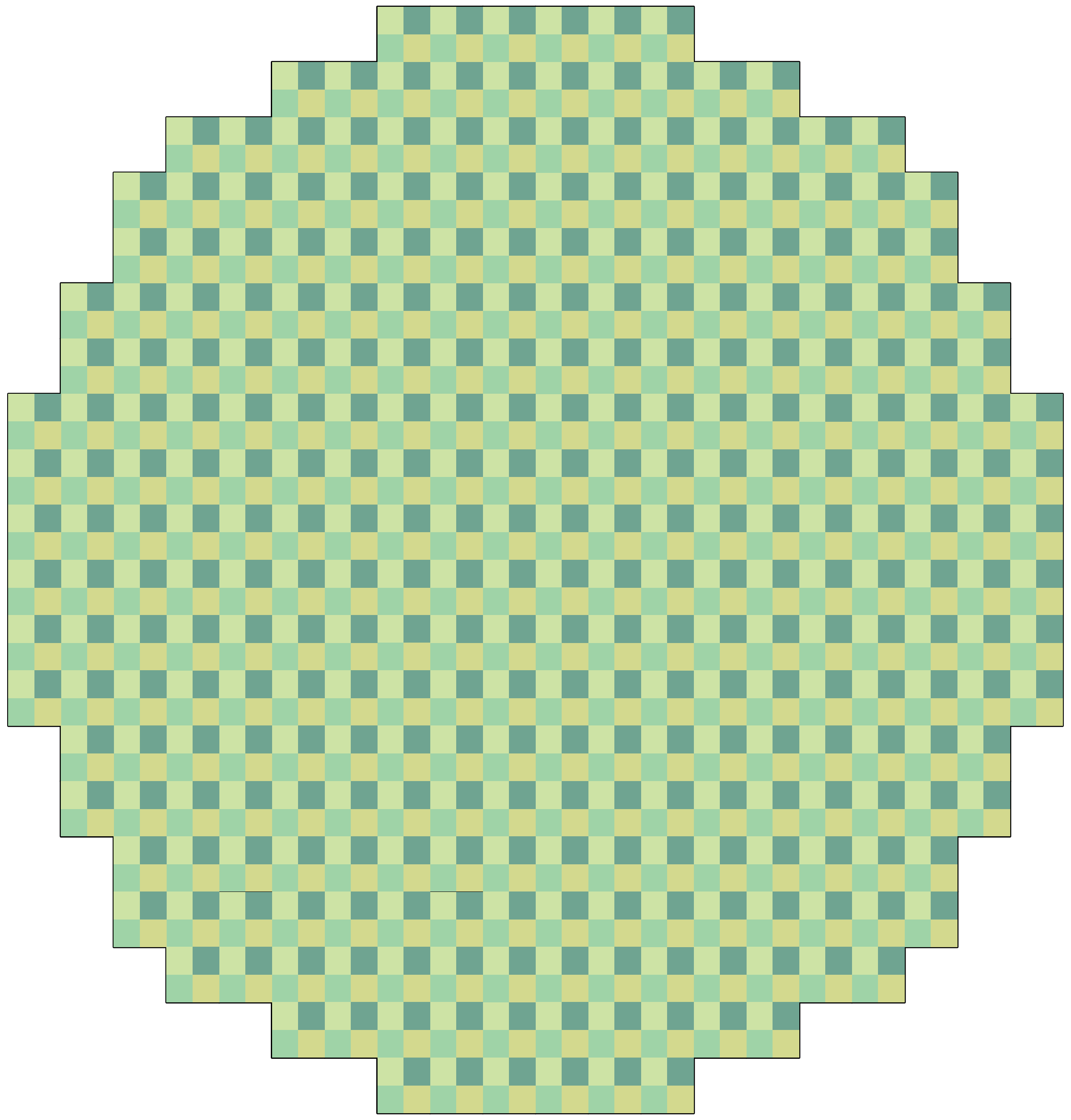
Ivan Ukhov, Mattias Villani, Petru Eles, and Zebo Peng

Linköping University

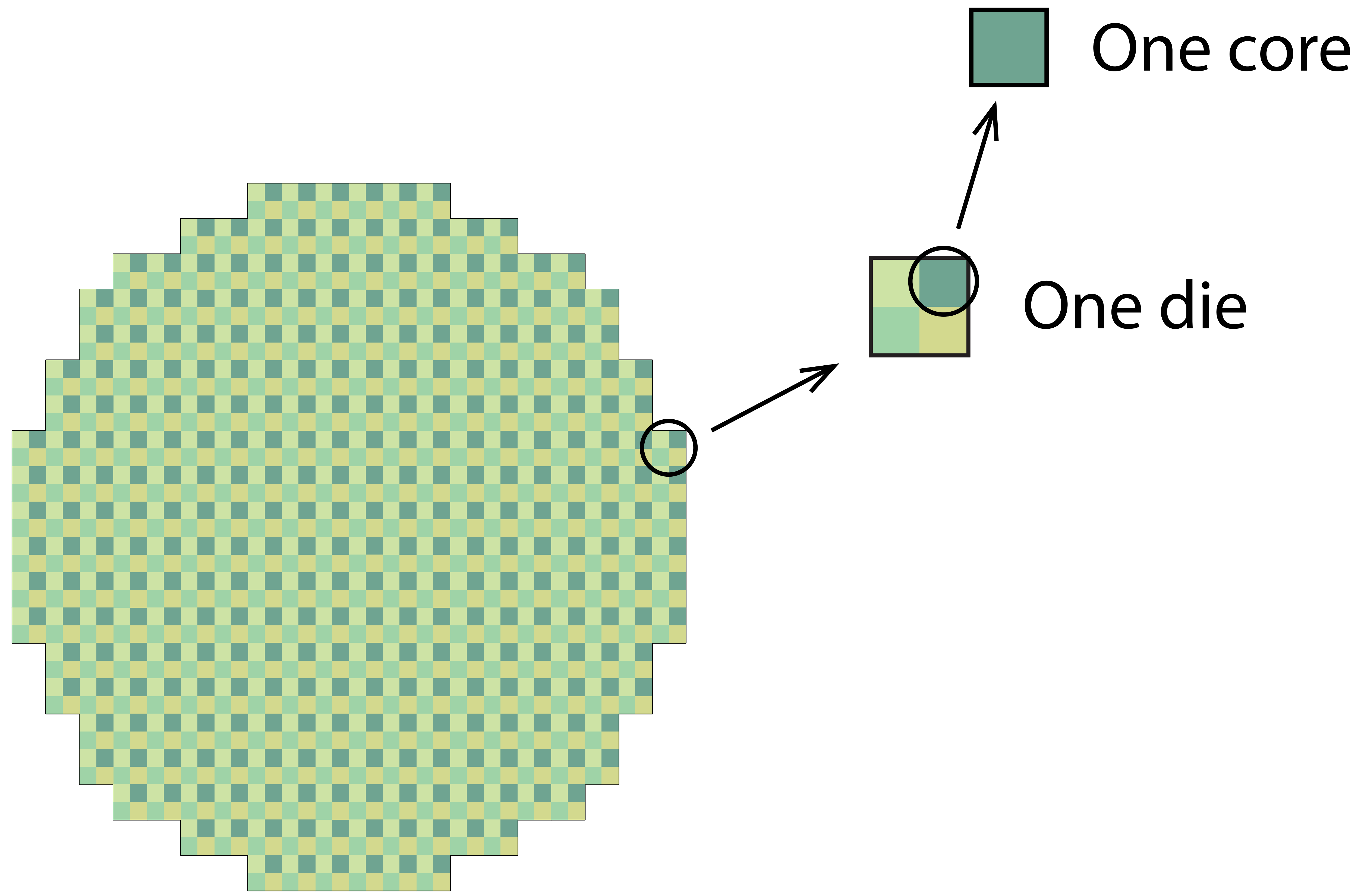
January 2014

Outline

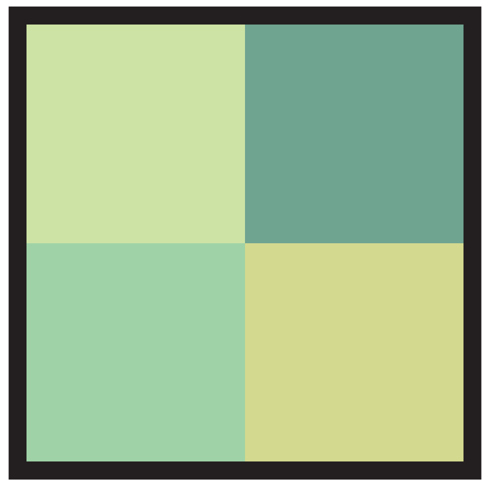
Wafer

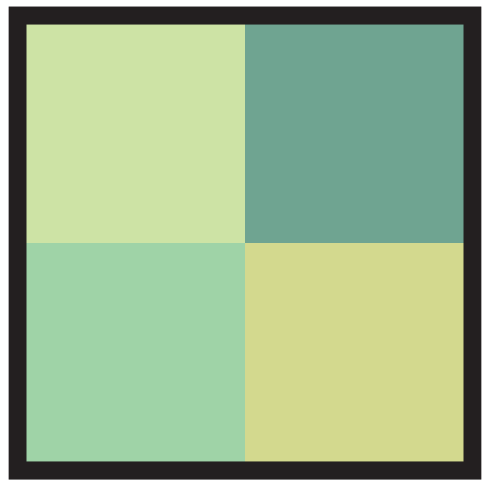


Wafer

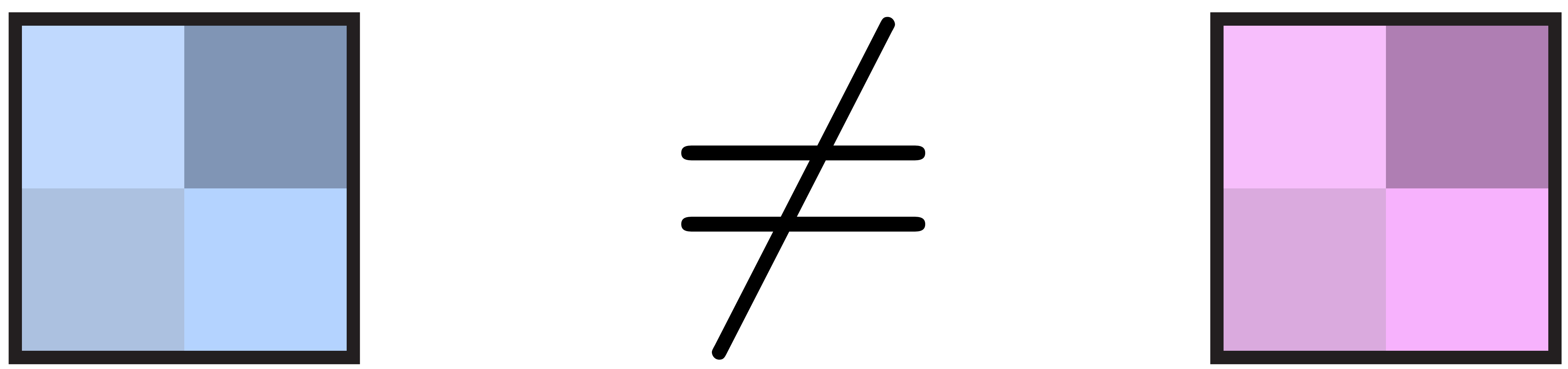


Process Variation

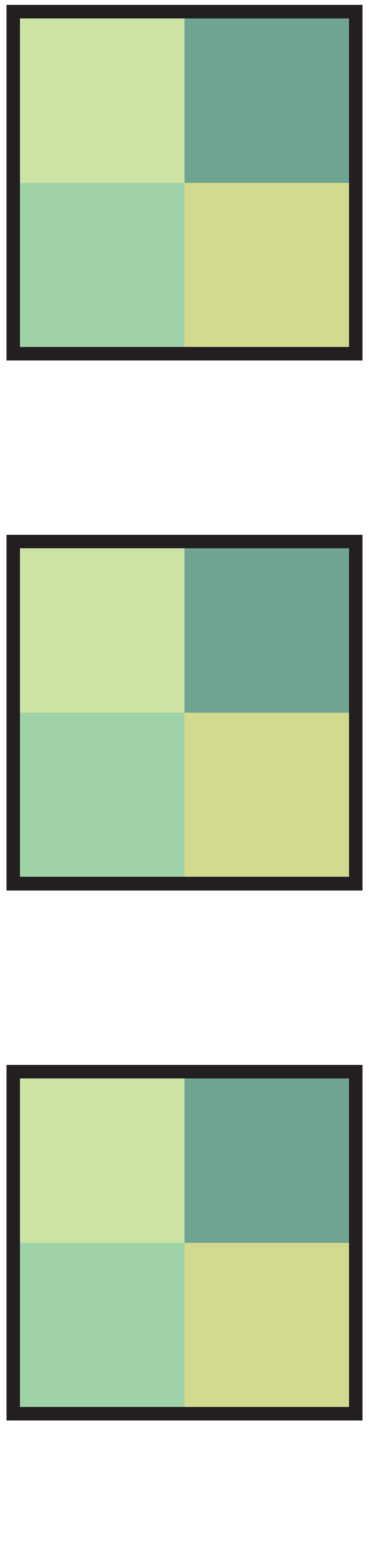




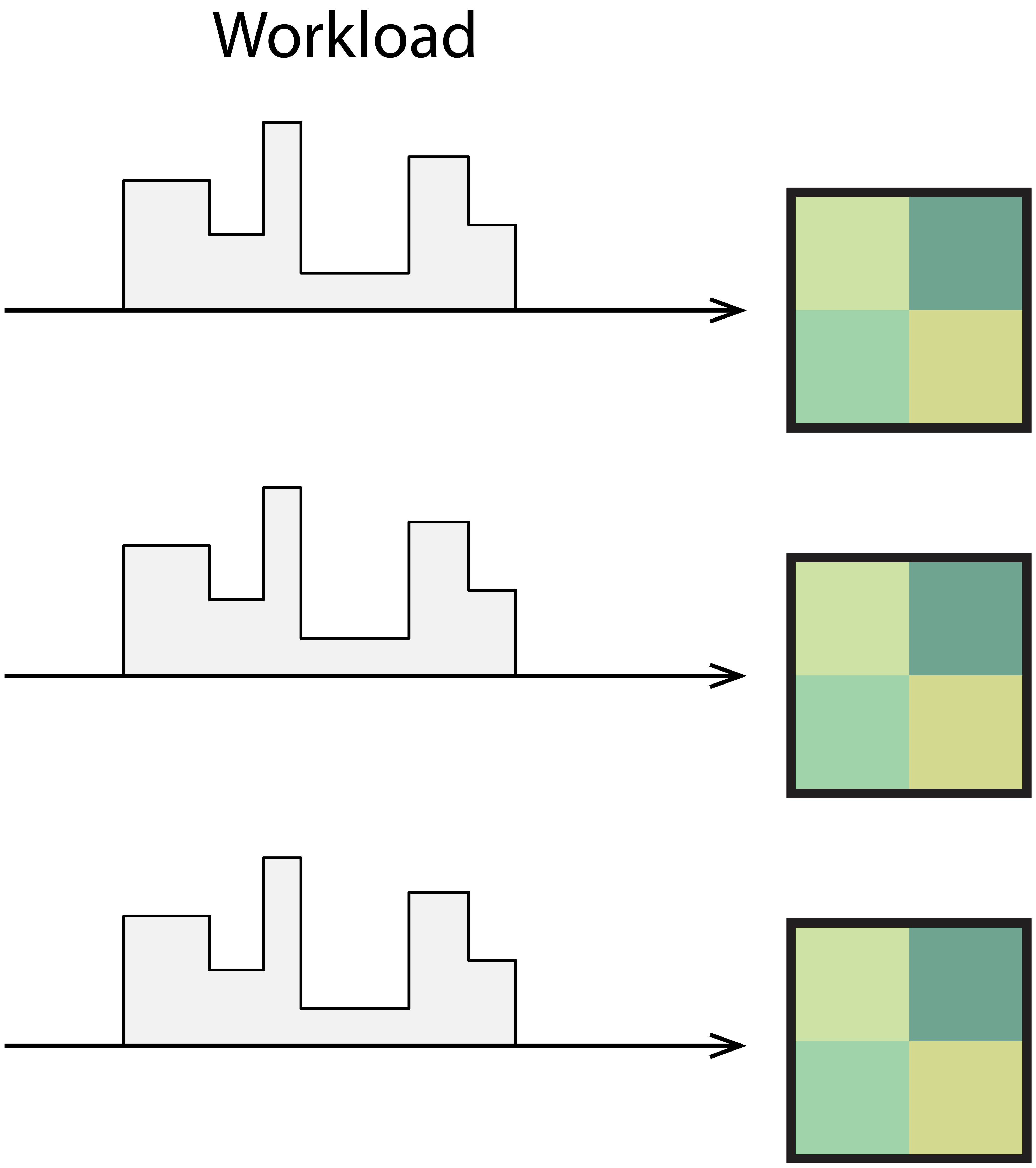
Process Variation



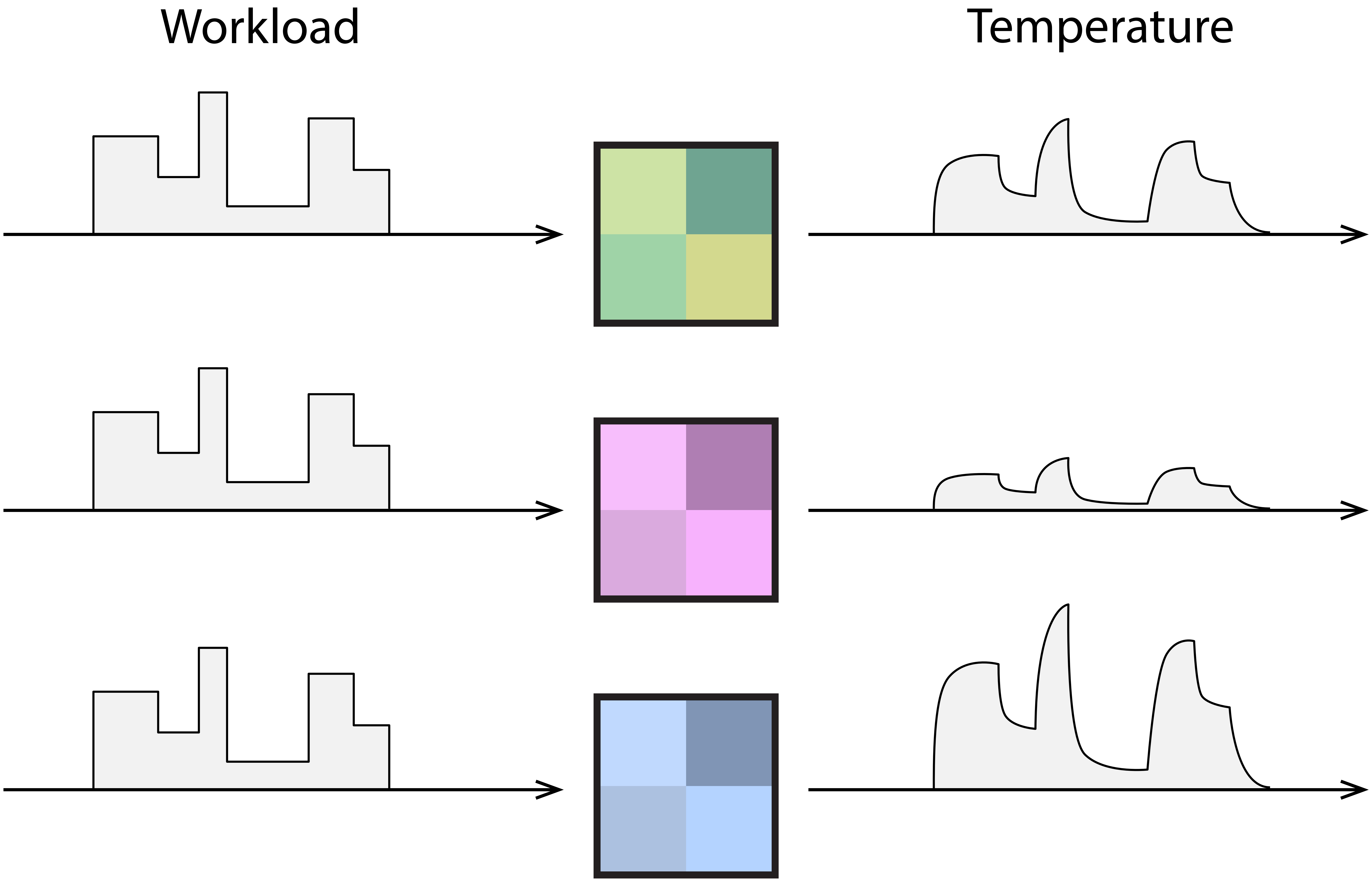
Process Variation



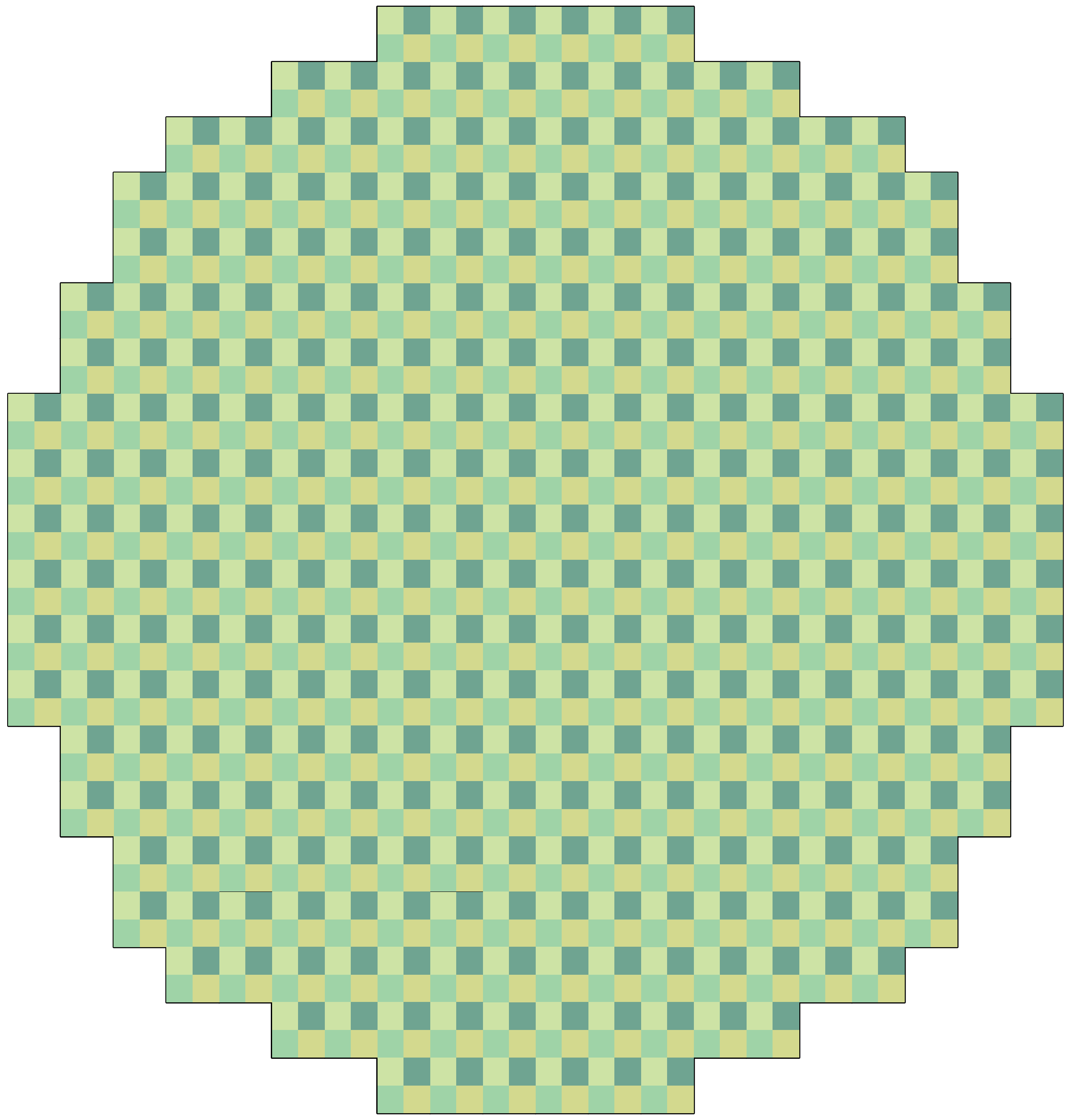
Process Variation



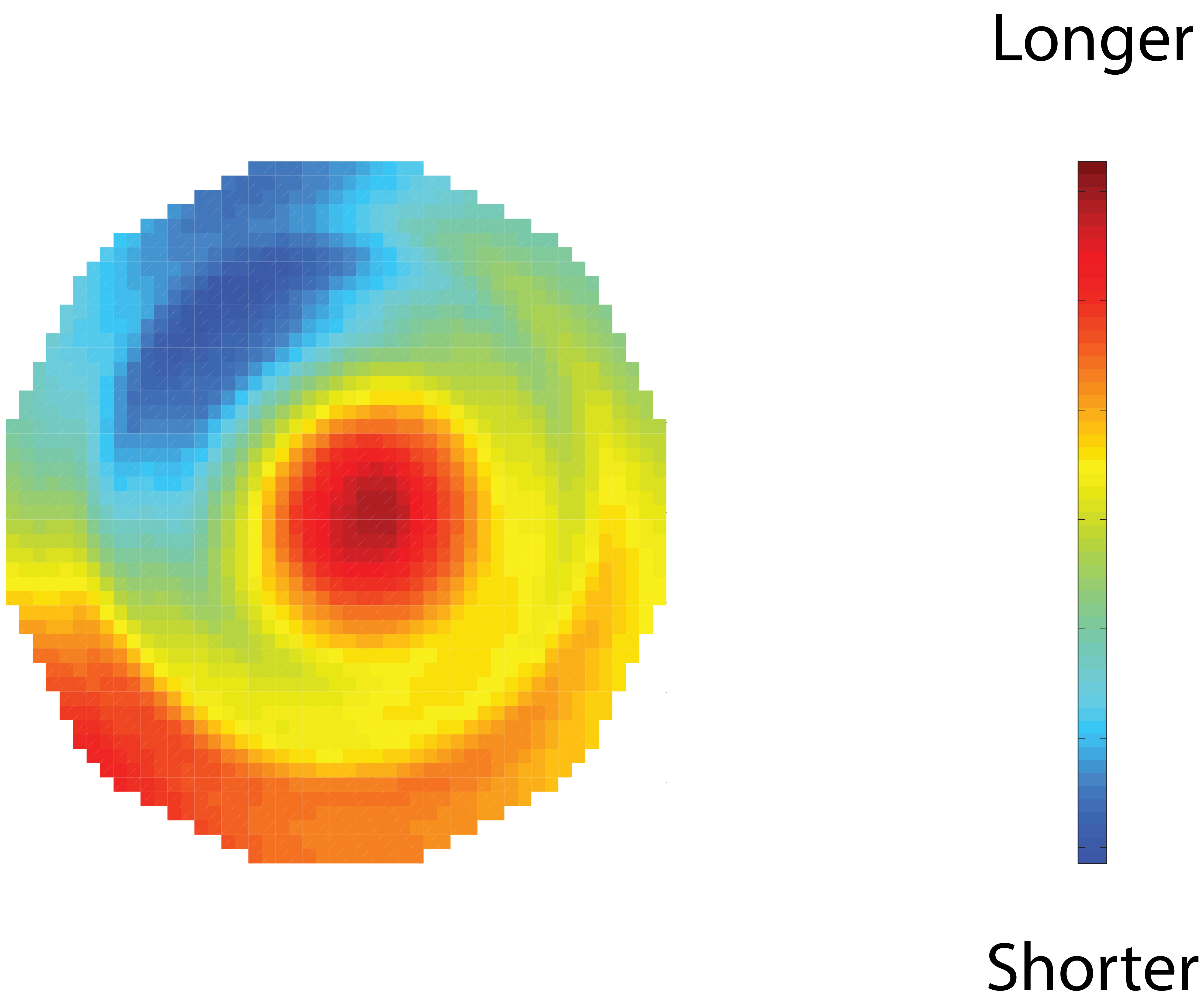
Process Variation



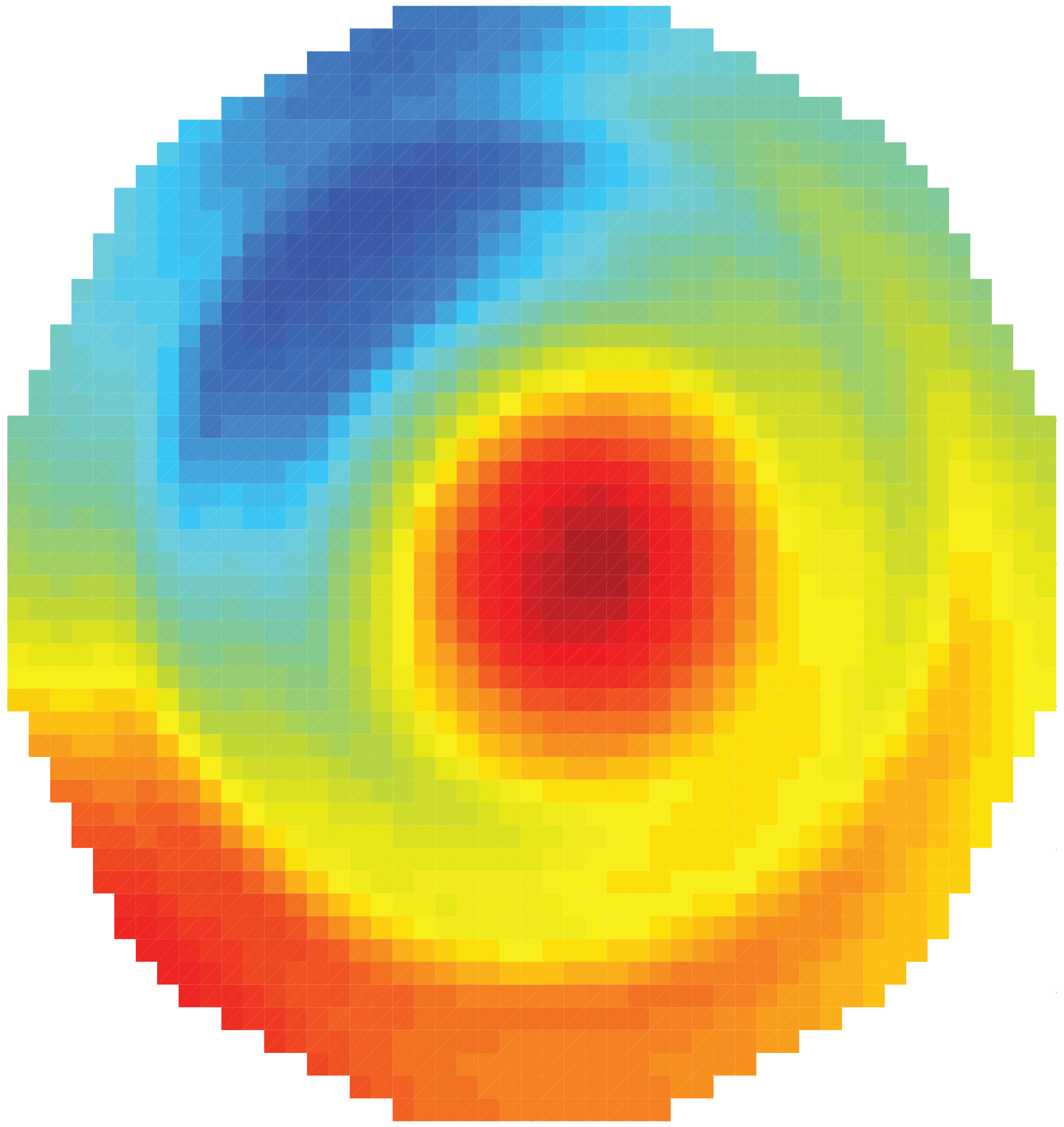
Process Variation



Process Variation



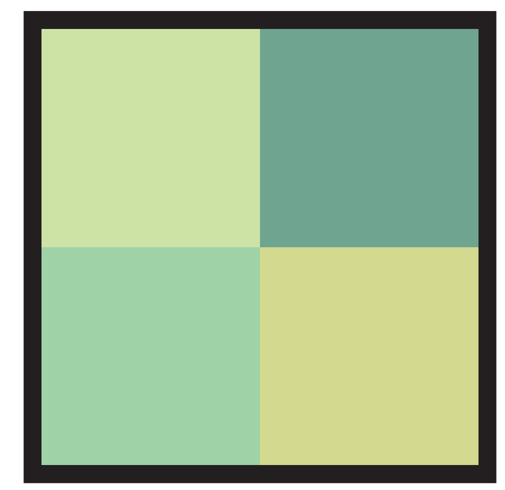
Our Goal



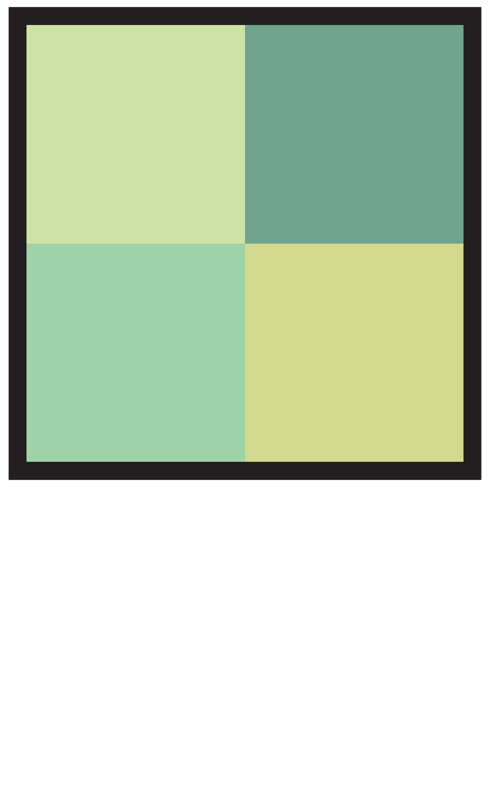
Our Goal

Quantity of interest

Alternatives

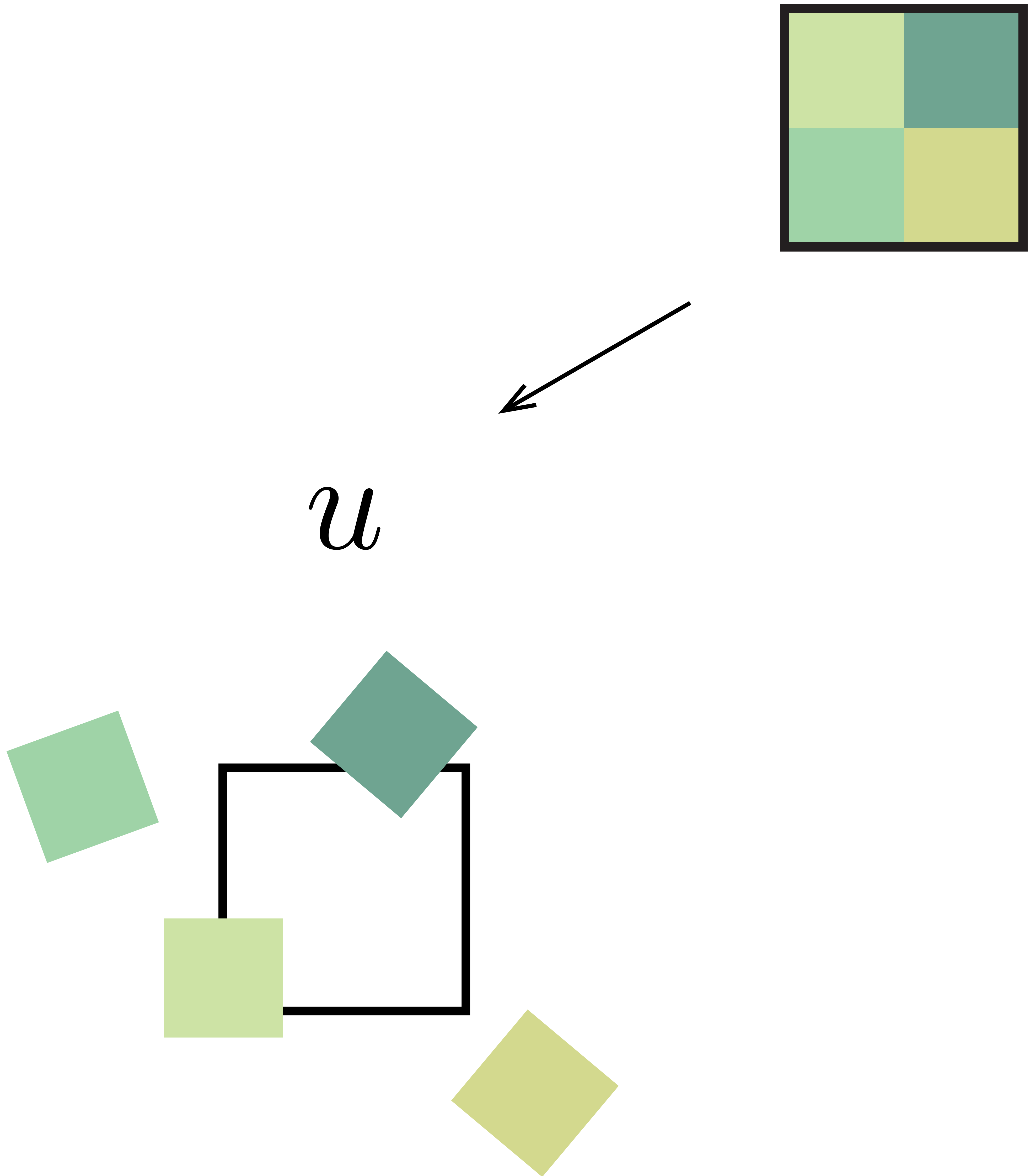


Alternatives

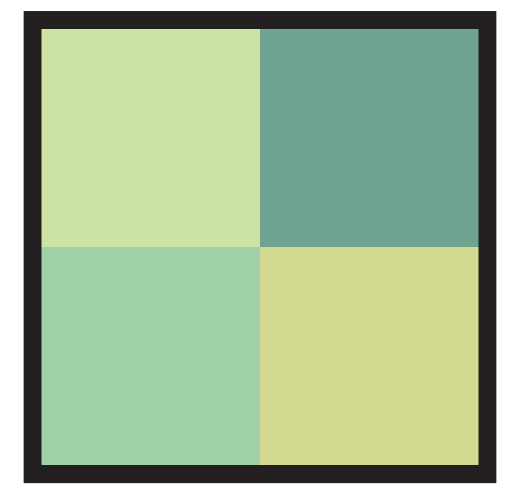


u

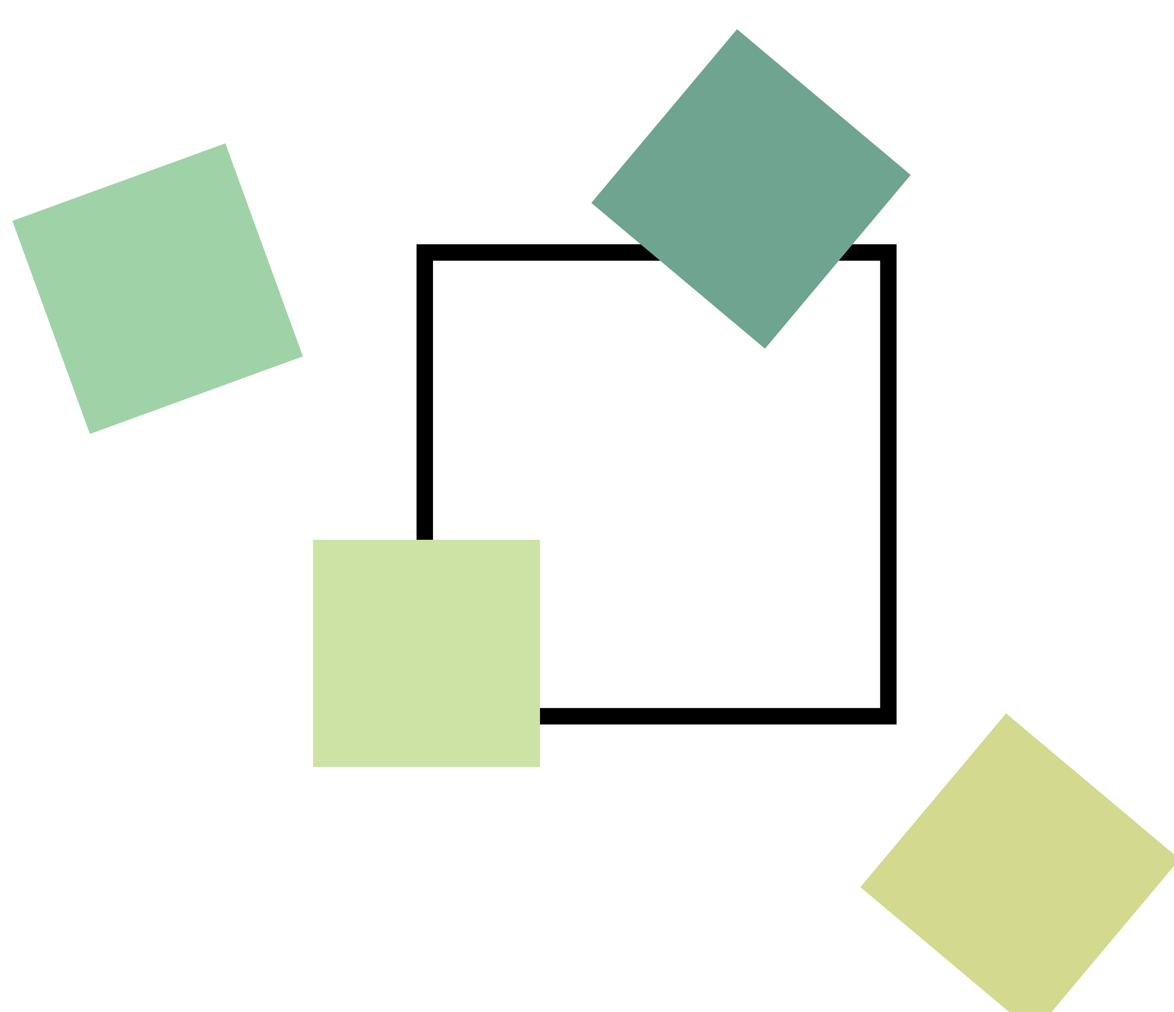
Alternatives



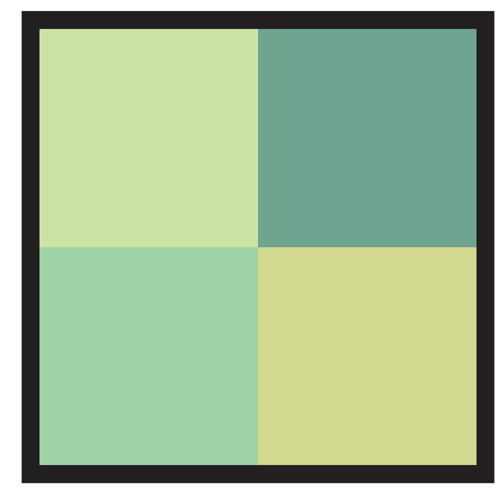
Alternatives



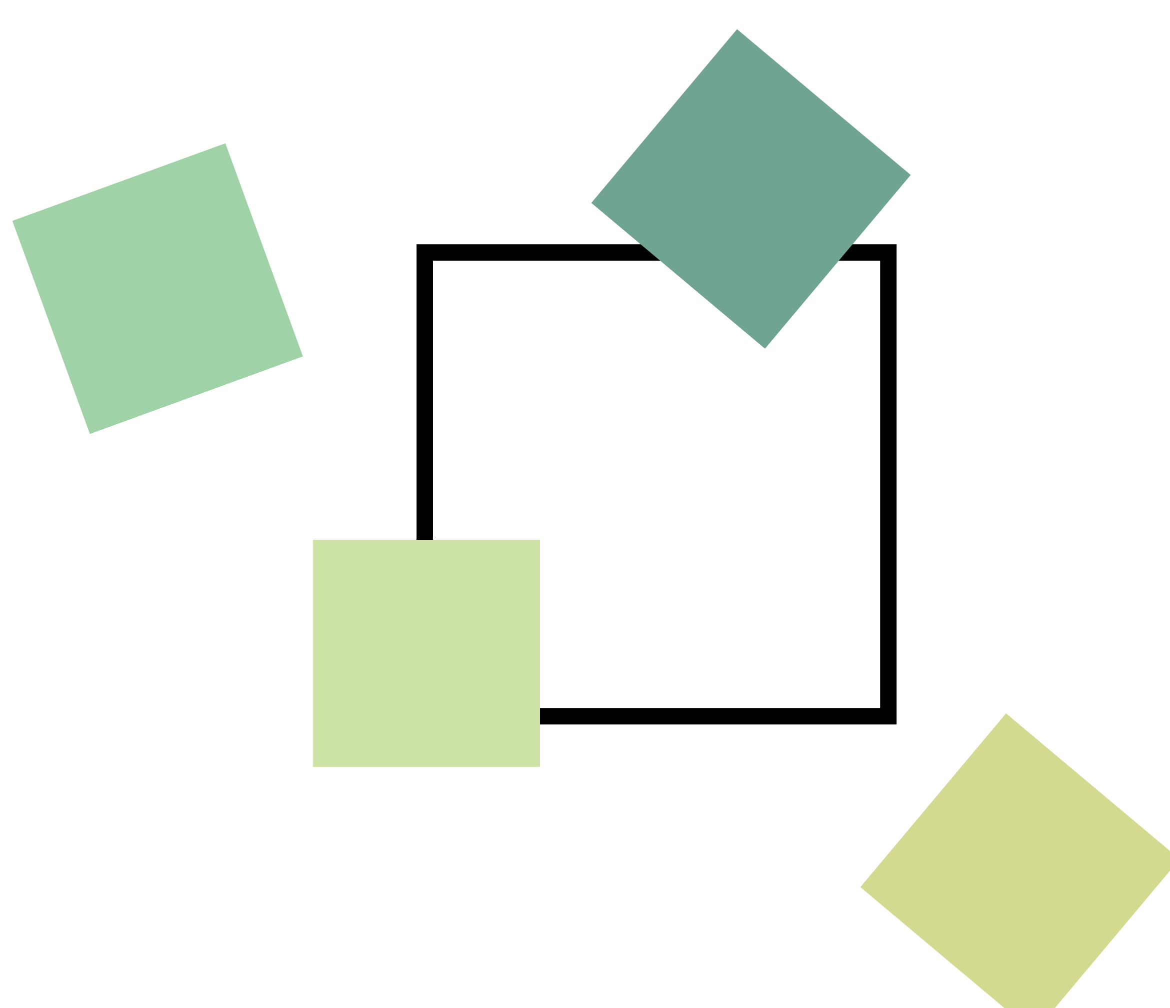
$$u \qquad\qquad q = h(u)$$



Alternatives

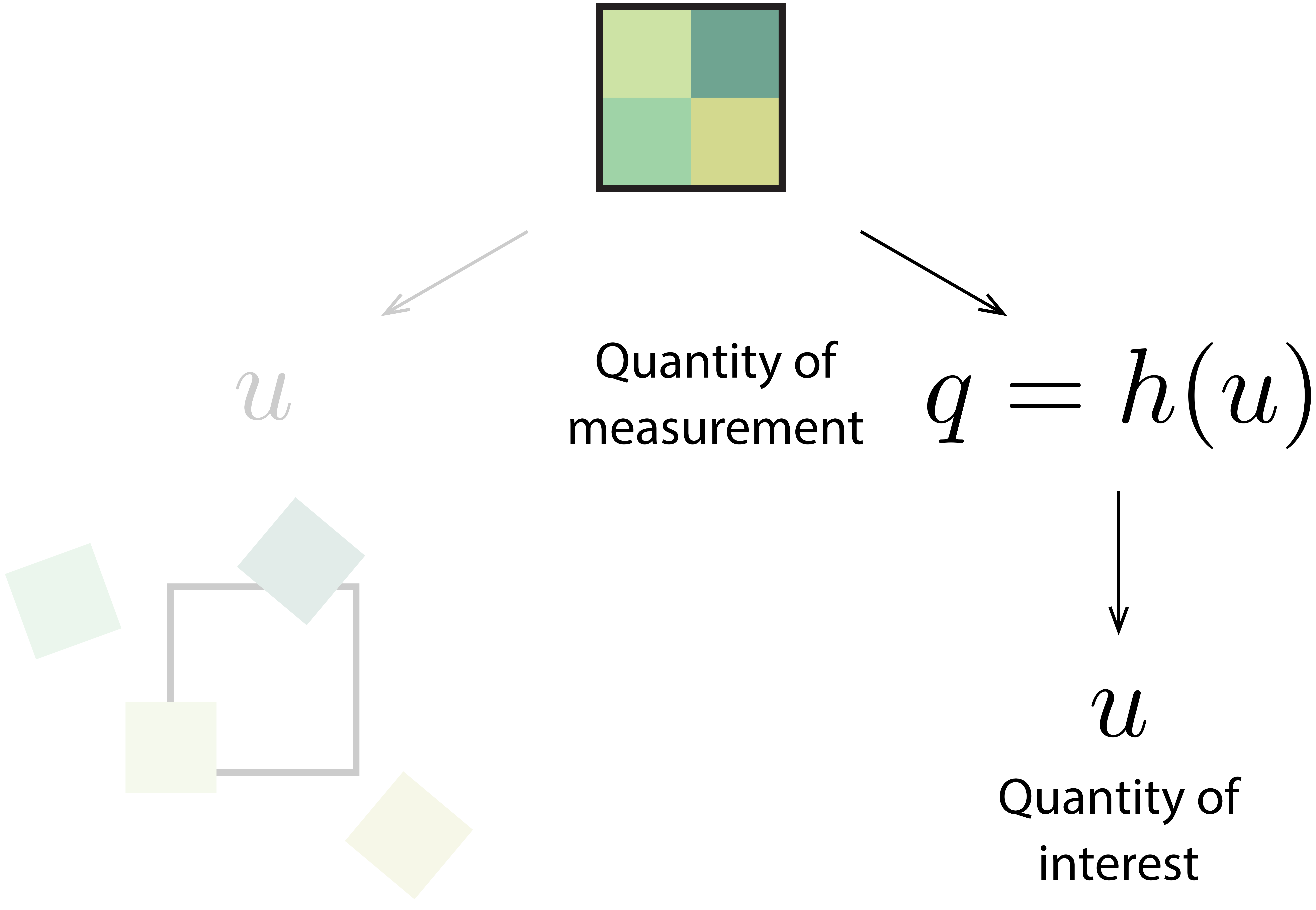


$$u \qquad\qquad q = h(u)$$



$$u?$$

Proposed Framework



Proposed Framework

$$\mathcal{Q} \longrightarrow \boxed{} \longrightarrow u$$

Proposed Framework

$$\mathcal{Q} \longrightarrow \boxed{} \longrightarrow u$$

Indirect

Proposed Framework

$$\mathcal{Q} \longrightarrow \boxed{} \longrightarrow u$$

Indirect

Incomplete

Proposed Framework

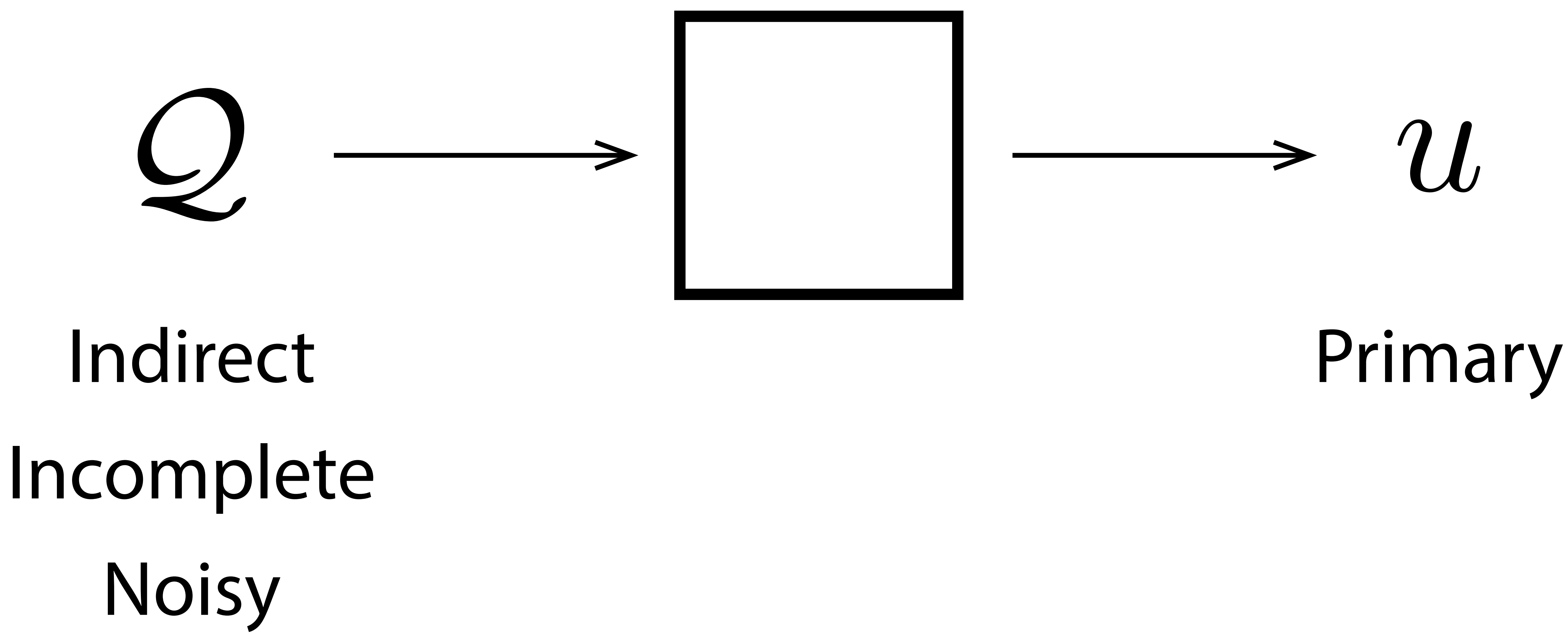
$$\mathcal{Q} \longrightarrow \boxed{} \longrightarrow u$$

Indirect

Incomplete

Noisy

Proposed Framework



Proposed Framework

$$\mathcal{Q} \longrightarrow \square \longrightarrow u$$

Indirect
Incomplete
Noisy

Primary
Comprehensive

Proposed Framework

$$\mathcal{Q} \longrightarrow \square \longrightarrow u$$

Indirect	Primary
Incomplete	Comprehensive
Noisy	Efficient

Illustrative Example

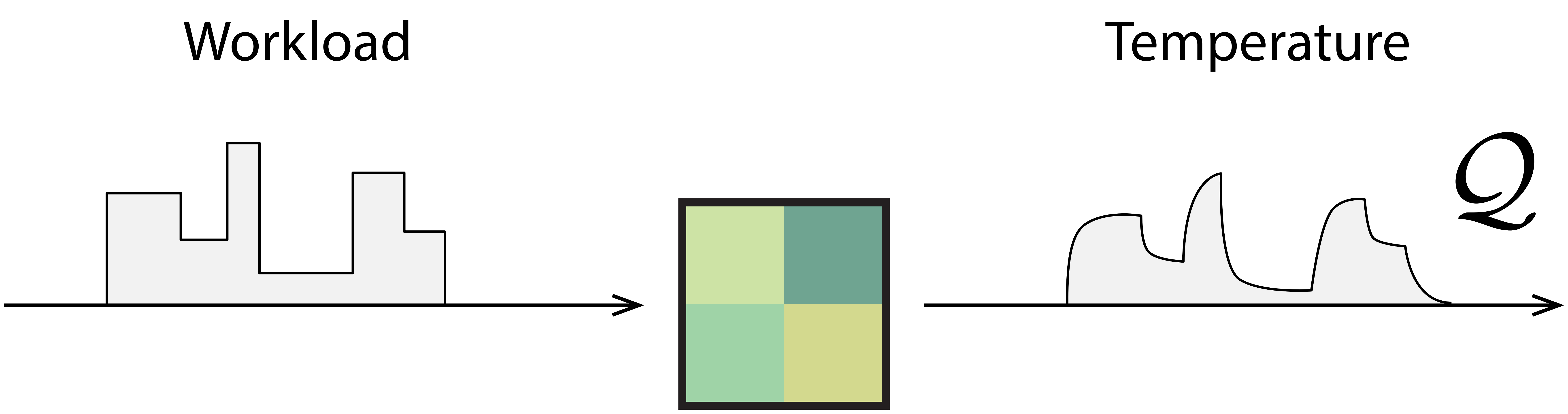
u Effective channel length

q Temperature

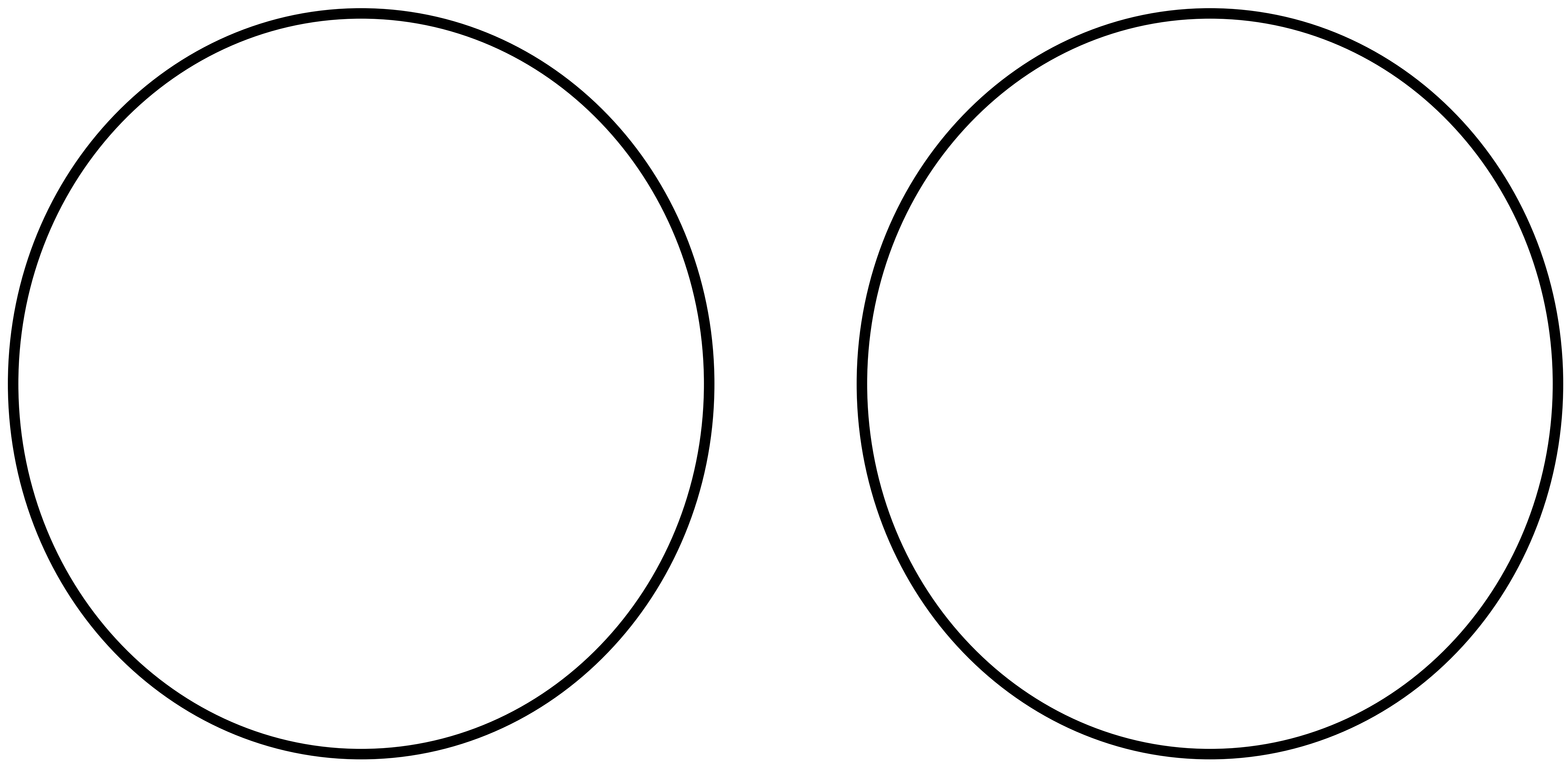
Illustrative Example

u Effective channel length

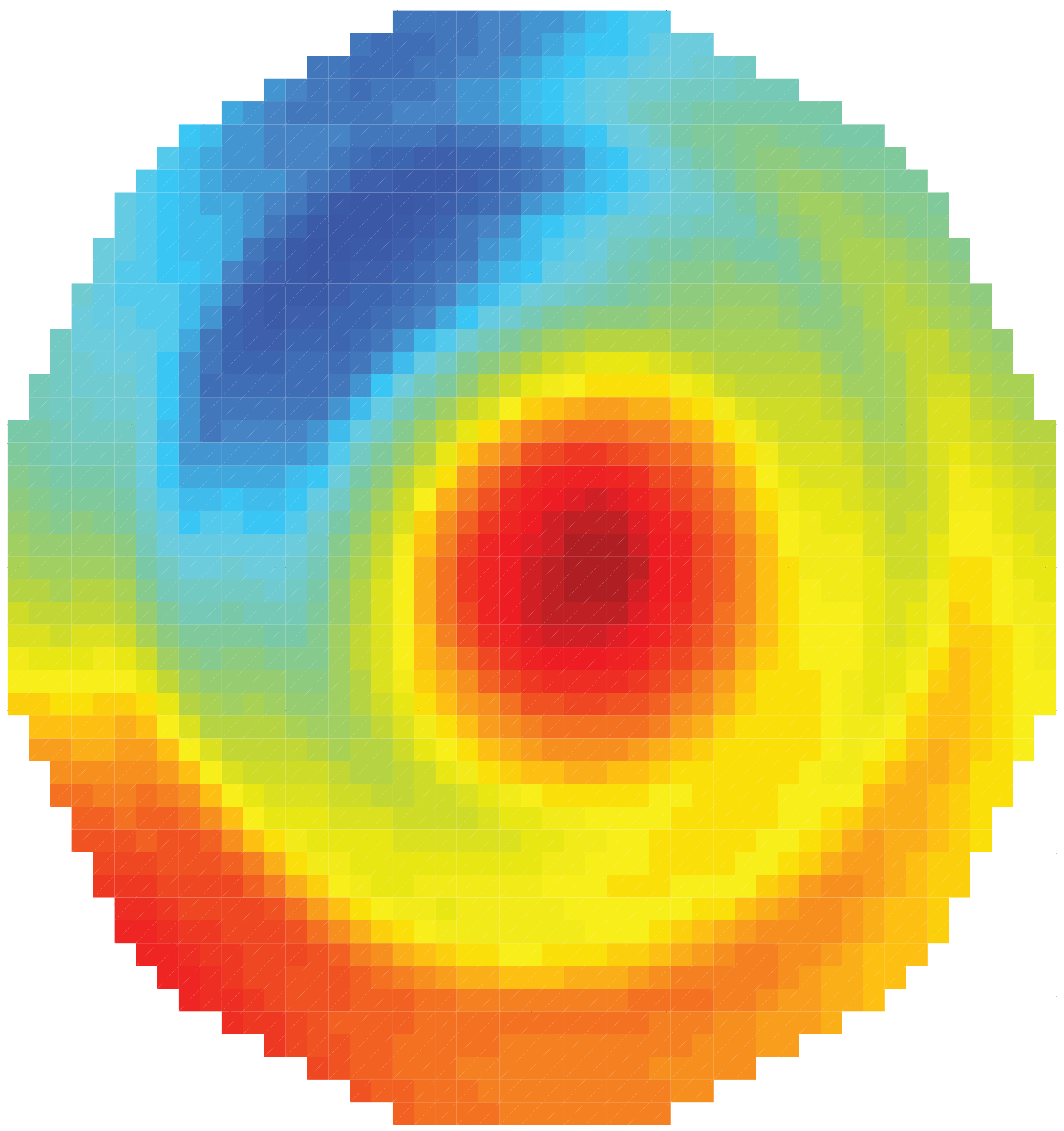
q Temperature

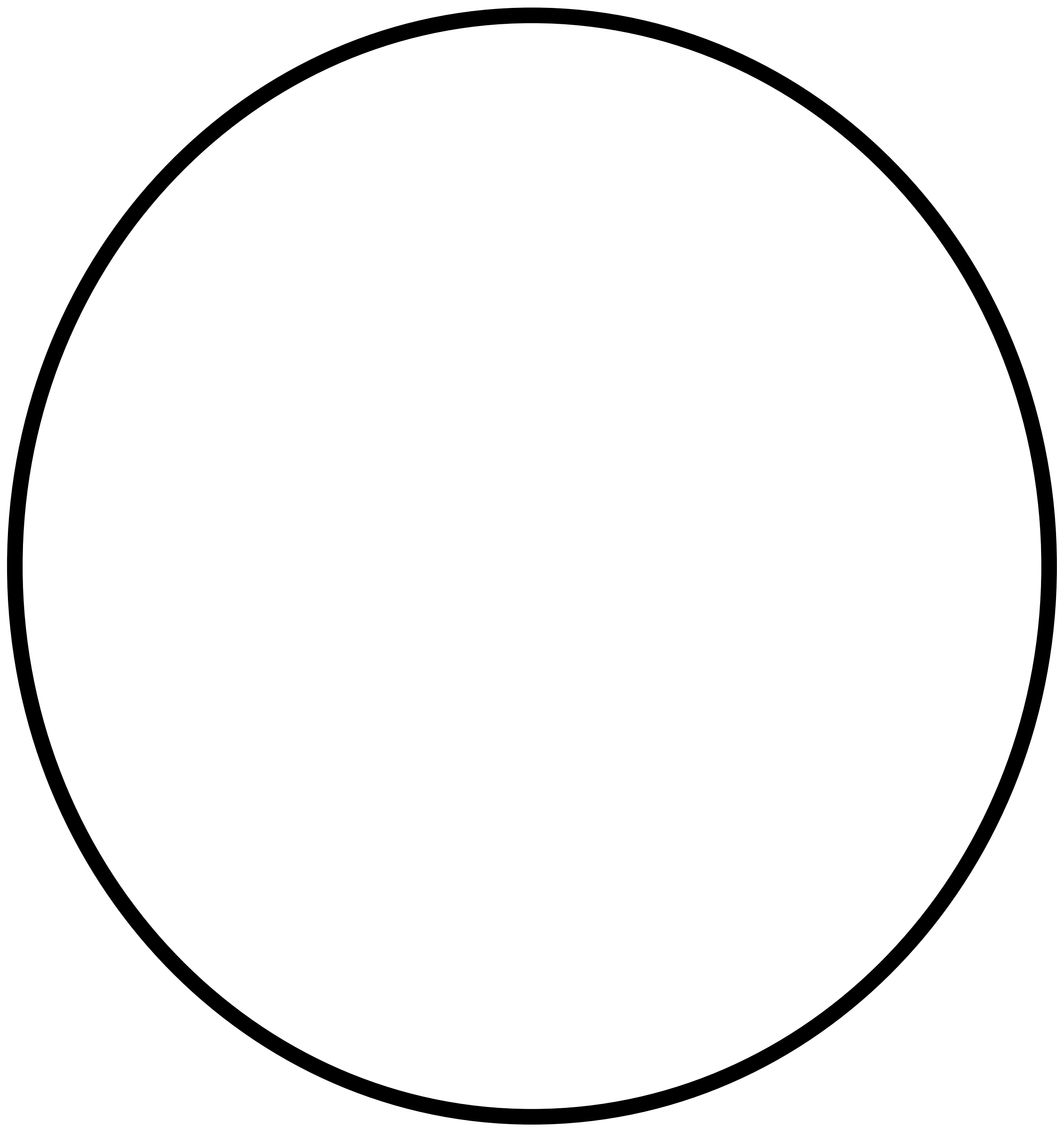


Illustrative Example



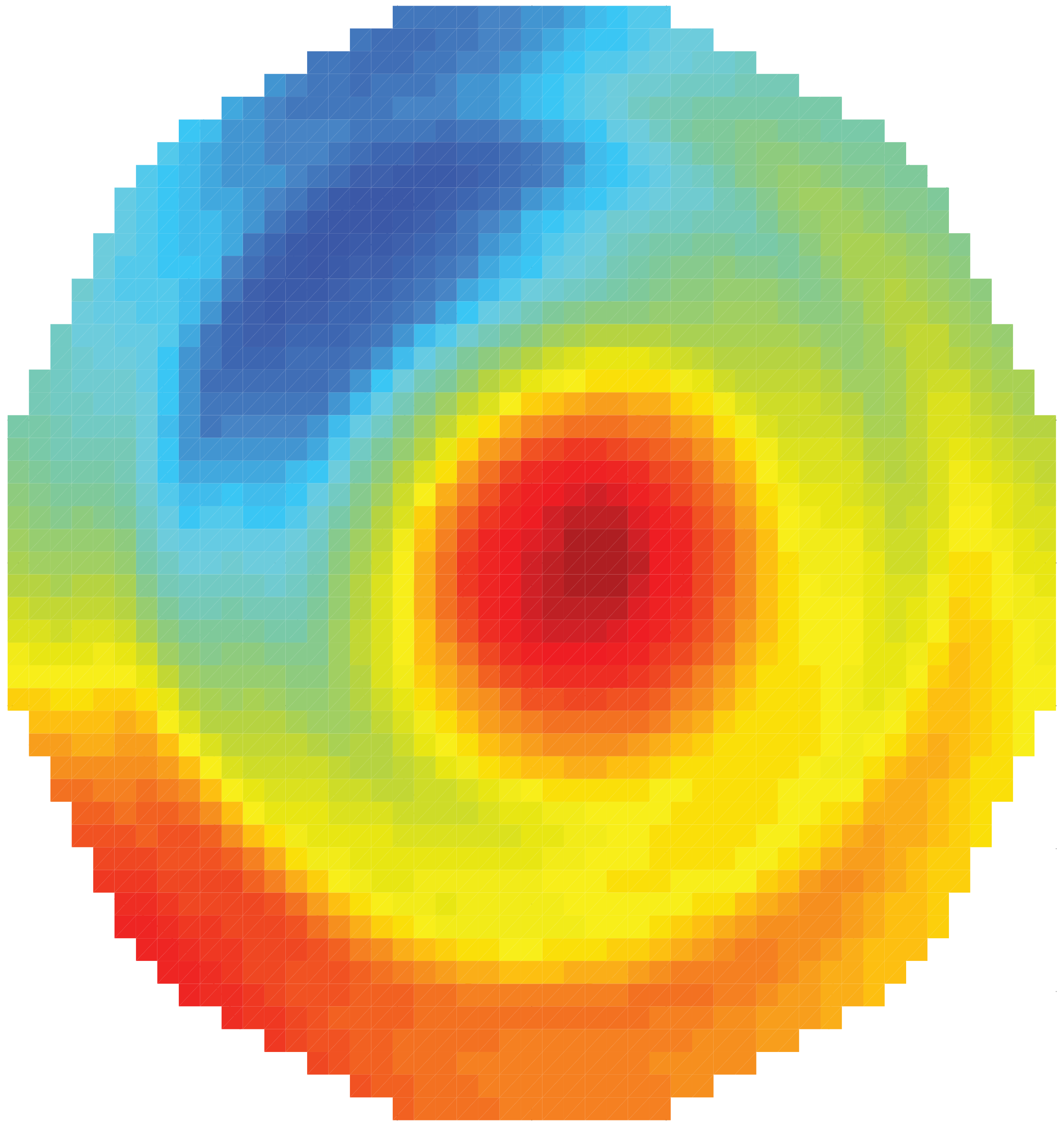
Illustrative Example



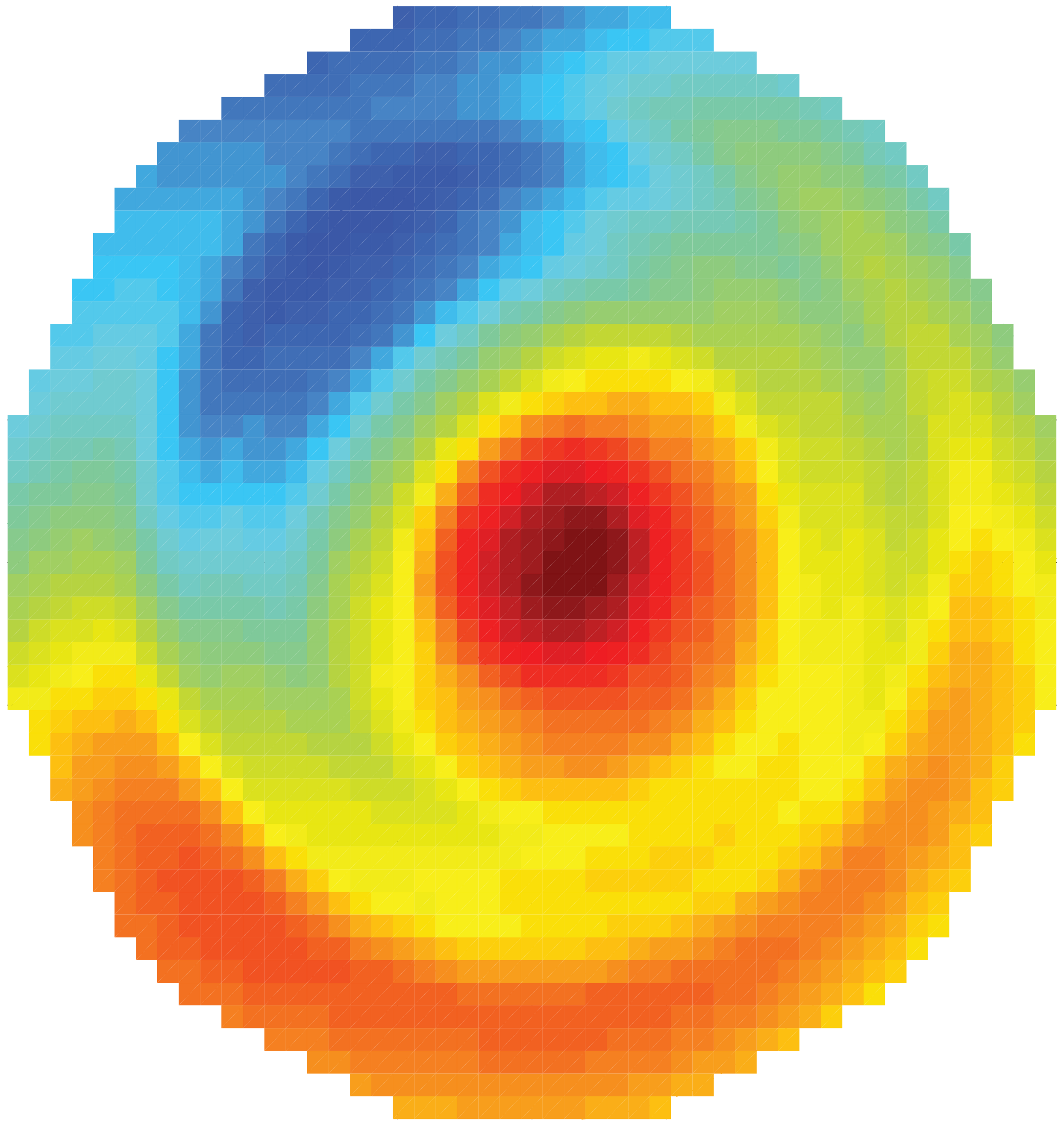


True

Illustrative Example

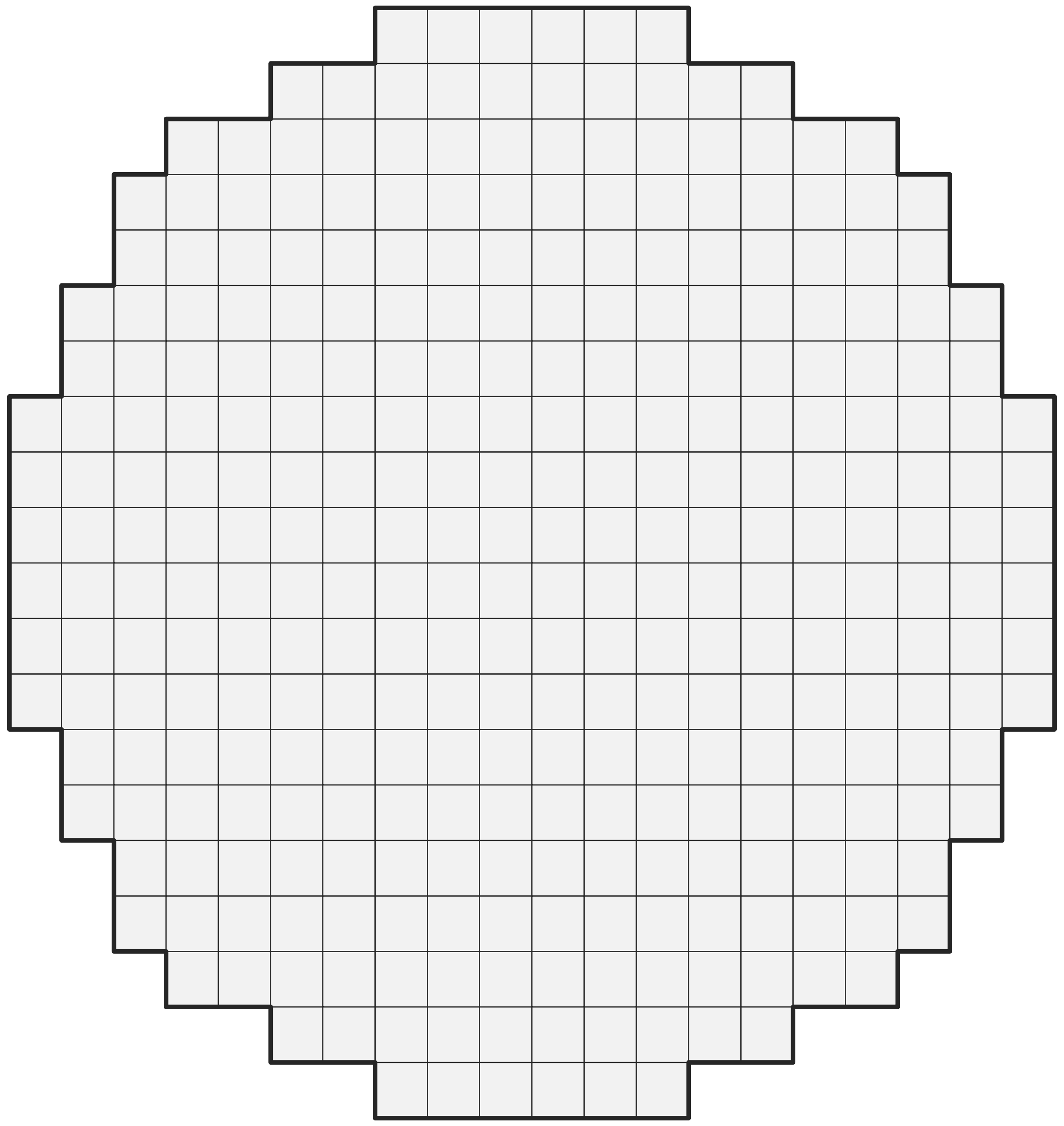


True



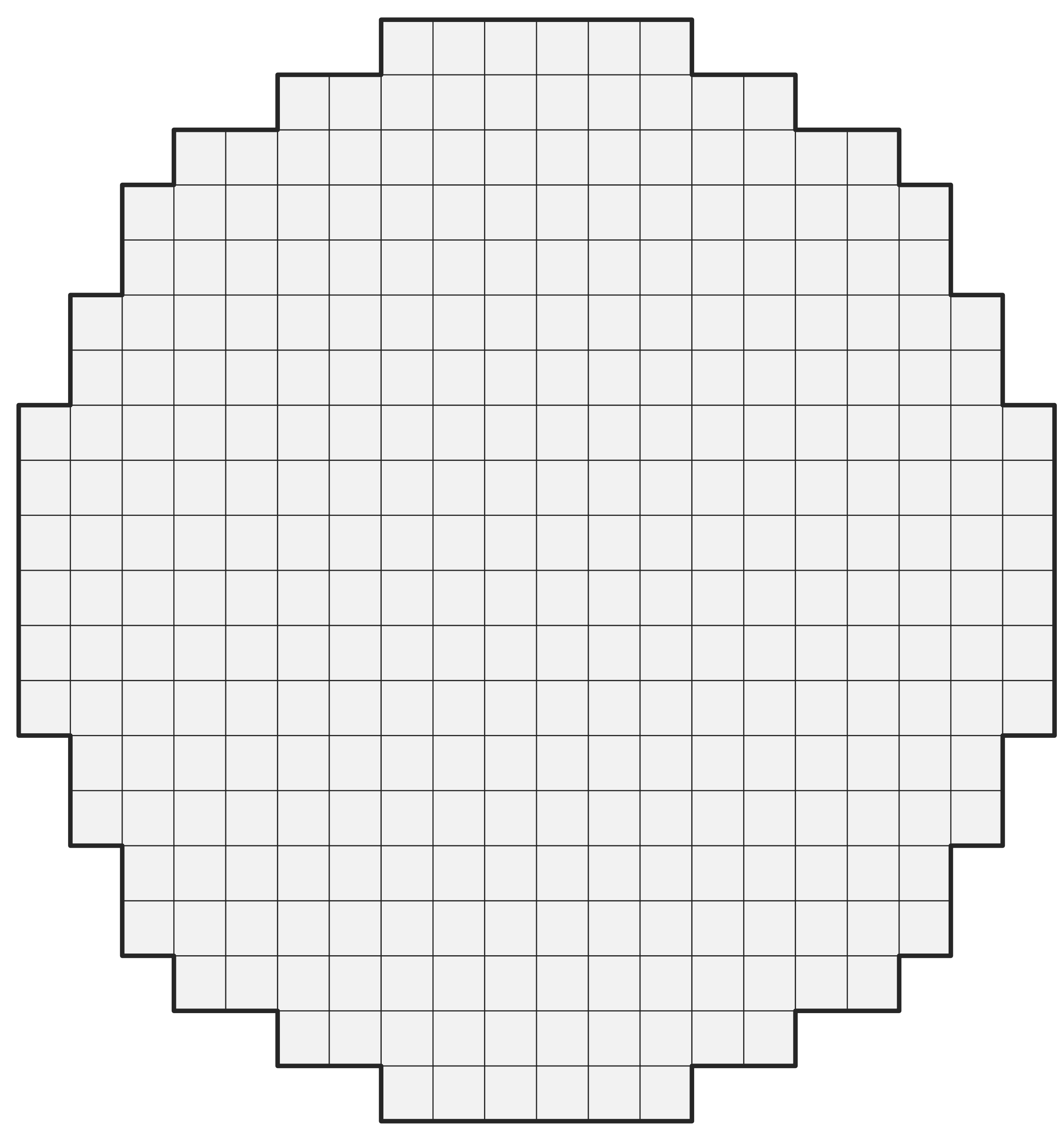
Inferred

Decision Making



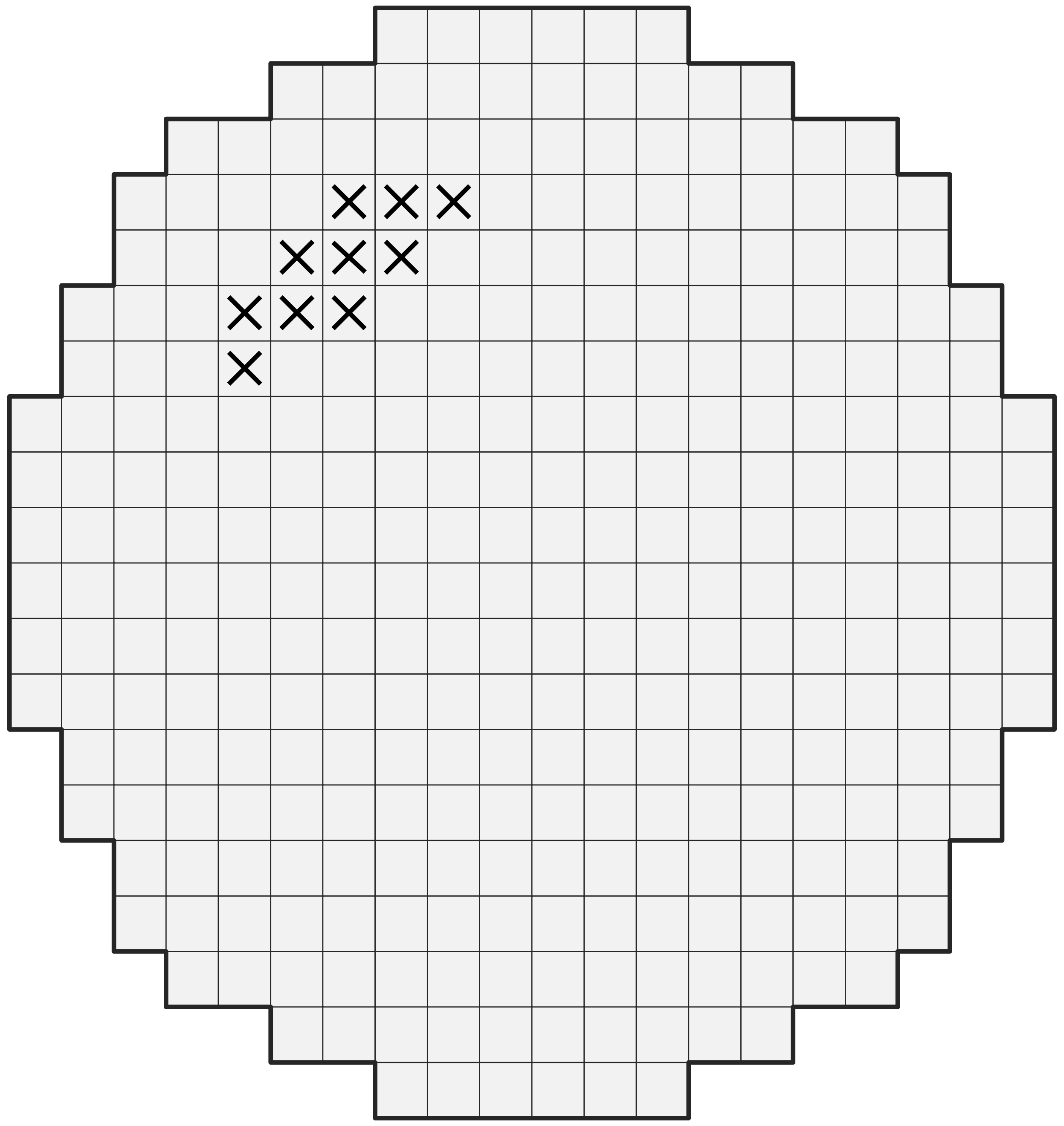
Decision Making

$$u > u_*$$



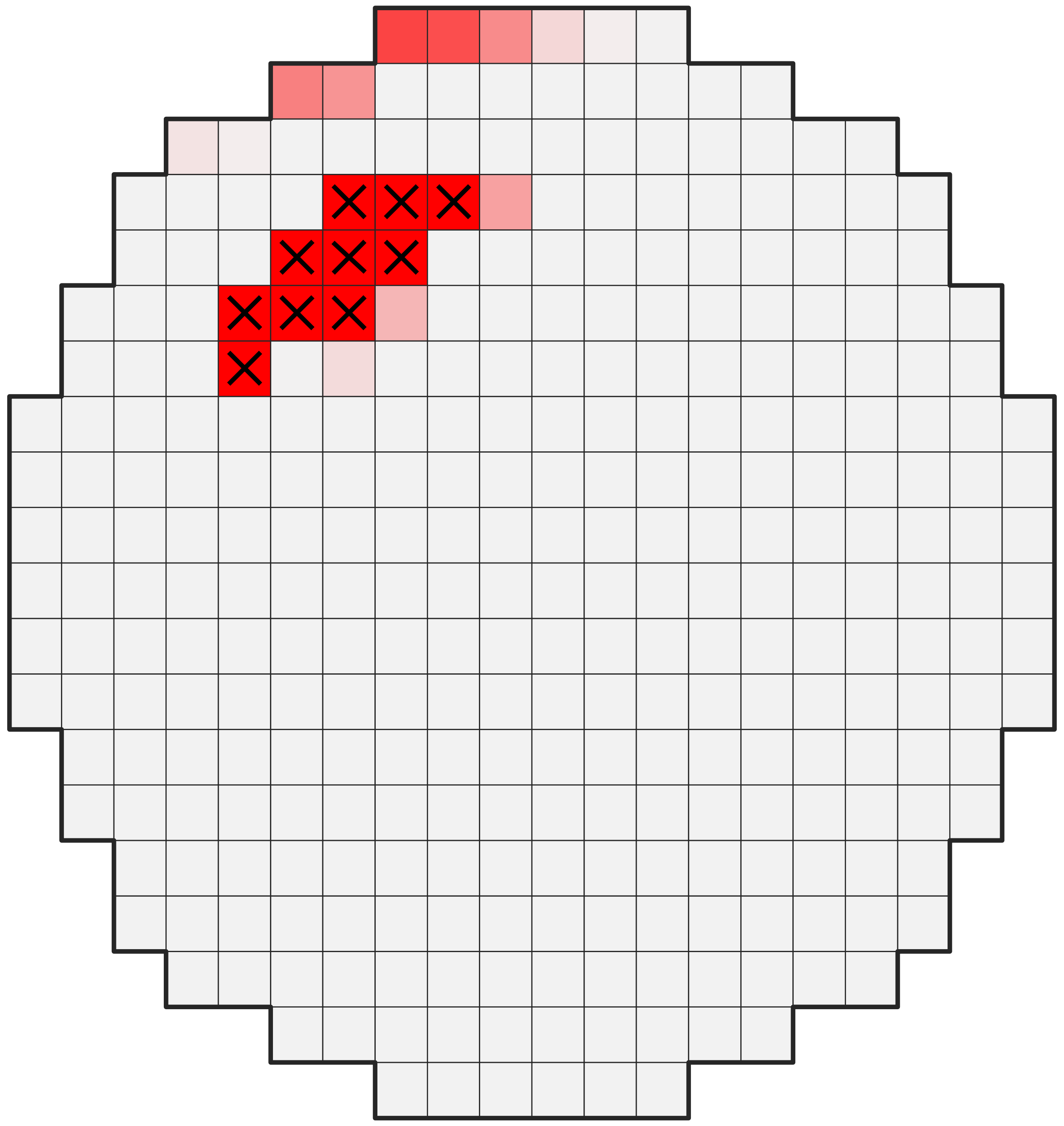
Decision Making

$$u < u_*$$

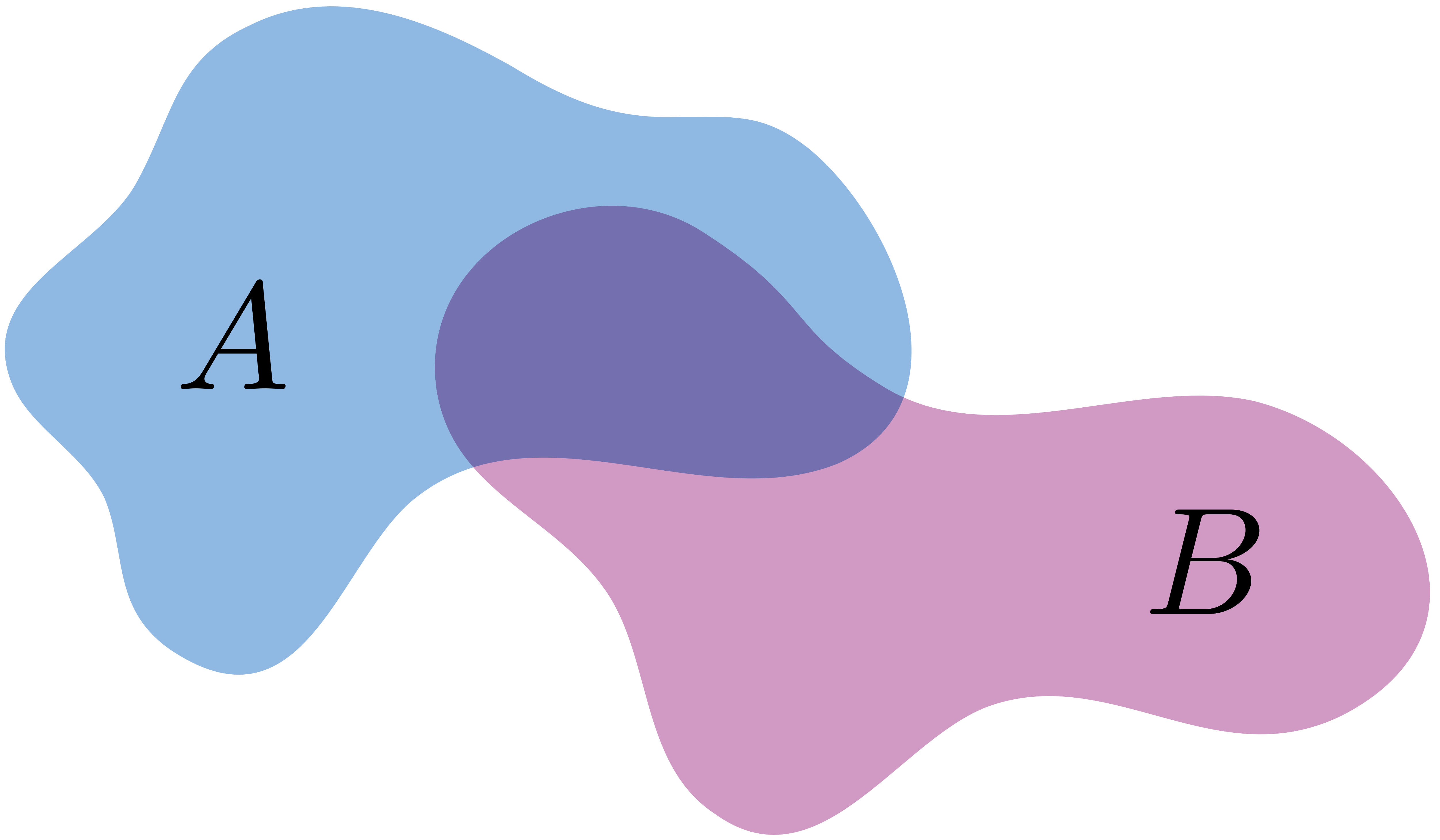


Decision Making

$$P(u < u_*)$$

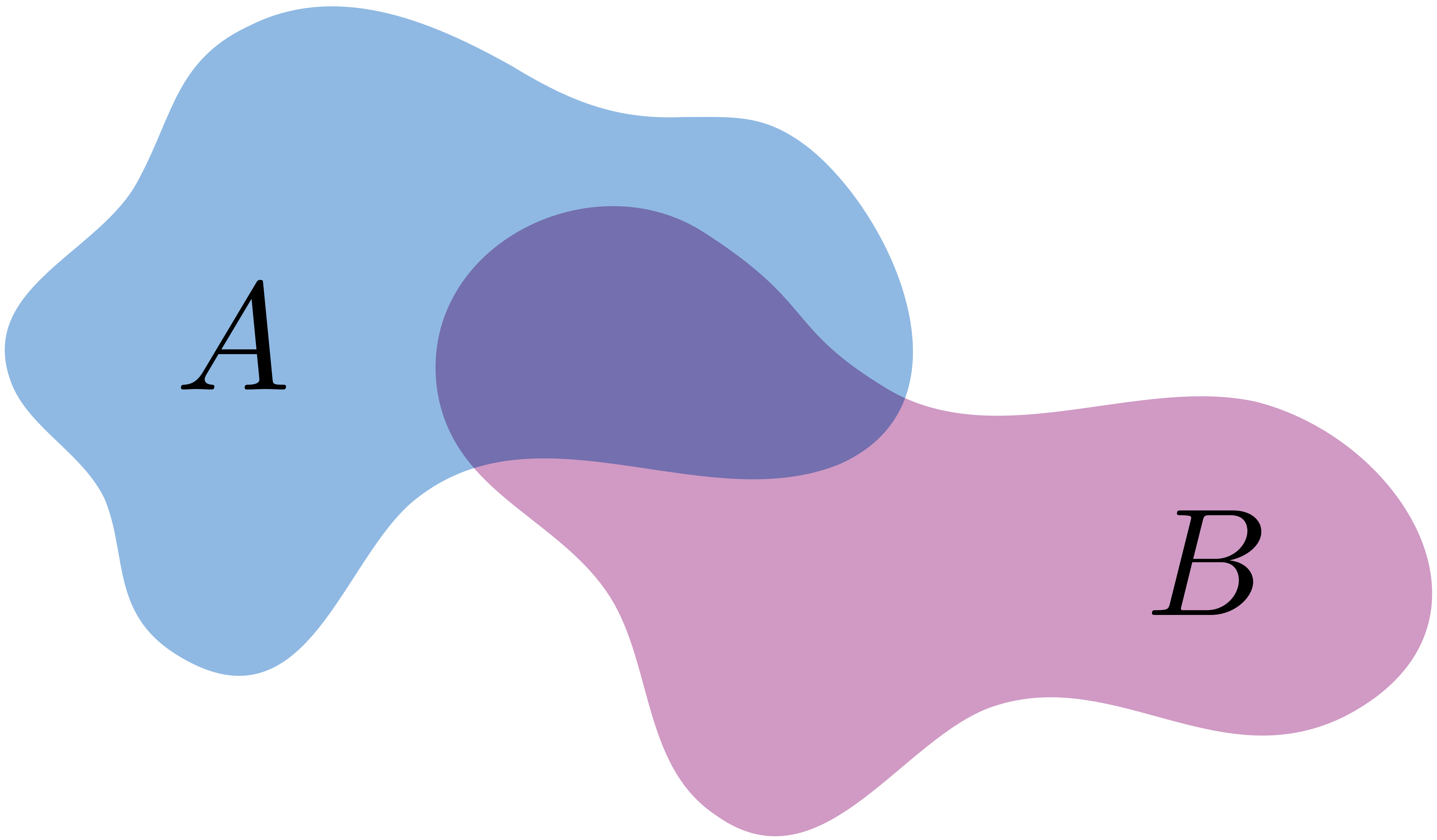


Bayesian Inference



Bayesian Inference

$$p(A|B) = ?$$



Bayesian Inference

$$p(A|B) \propto p(B|A) \times p(A)$$

Bayesian Inference

Posterior Likelihood Prior

$$p(A|B) \propto p(B|A) \times p(A)$$

Bayesian Inference

Posterior Likelihood Prior

$$p(u|\mathcal{Q}) \propto p(\mathcal{Q}|u) \times p(u)$$

Prior

$$p(u)$$

Prior

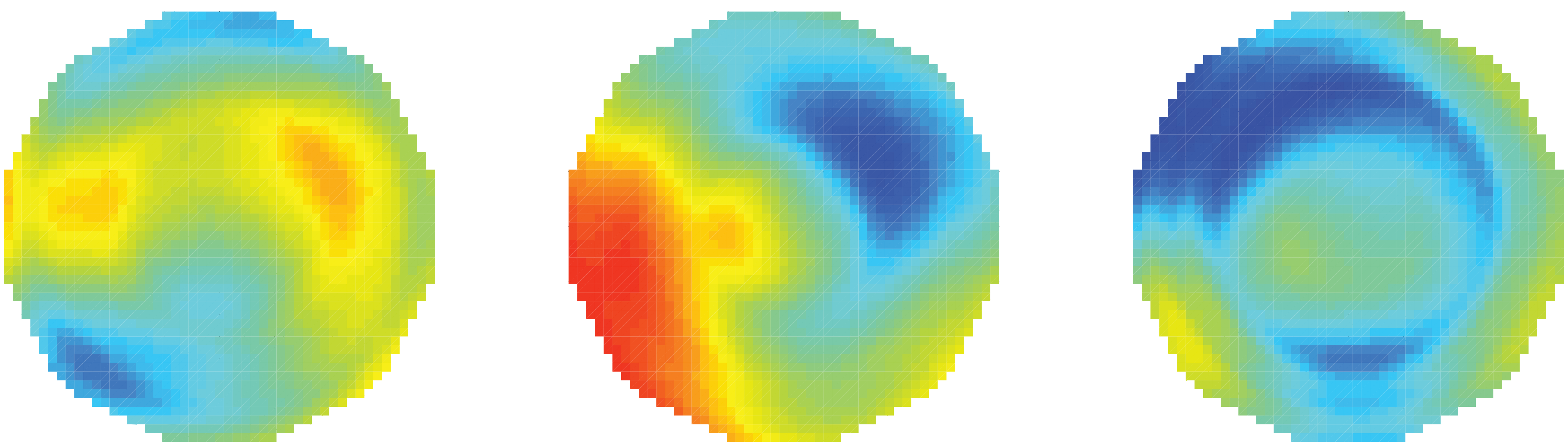
$$p(u)$$

$$u \sim \mathcal{GP}(\mu, k)$$

Prior

$$p(u)$$

$$u \sim \mathcal{GP}(\mu, k)$$



Likelihood

$$p(\mathcal{Q}|u)$$

Likelihood

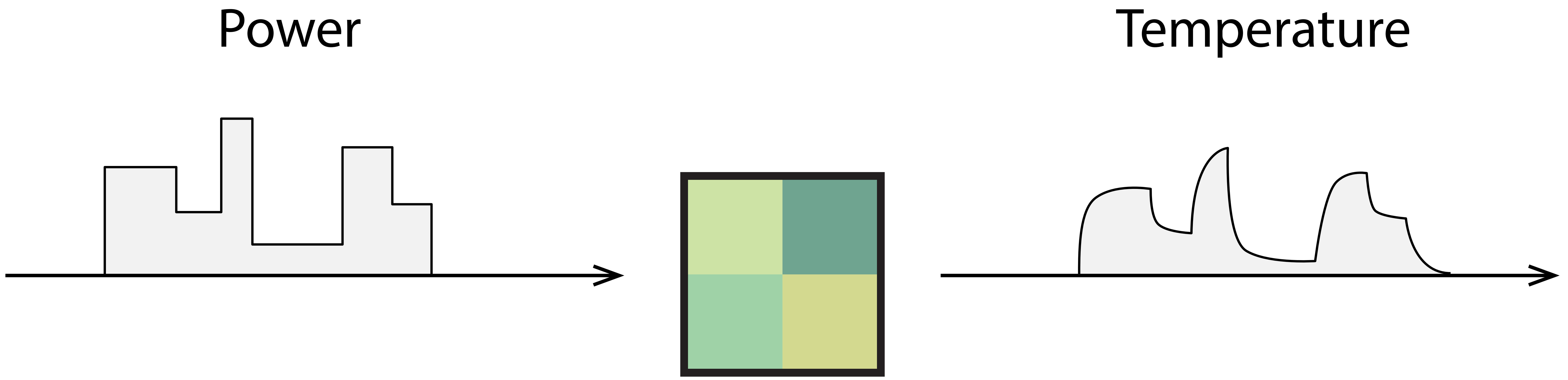
$$p(\mathcal{Q}|u)$$

$$q = f(u)$$

Likelihood

$$p(\mathcal{Q}|u)$$

$$q = f(u)$$

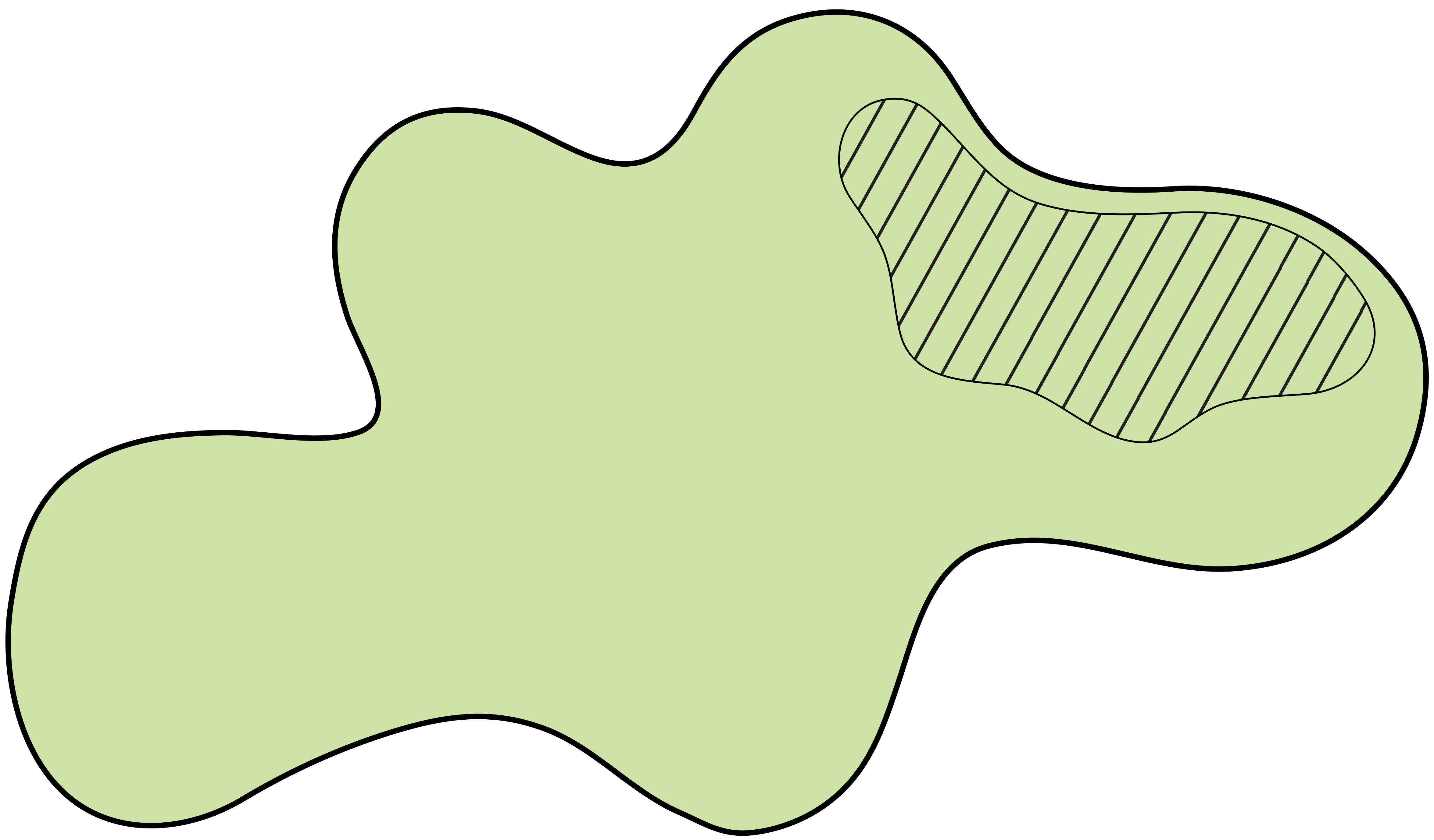


Posterior

$$p(u|\mathcal{Q}) \propto p(\mathcal{Q}|u) \times p(u)$$

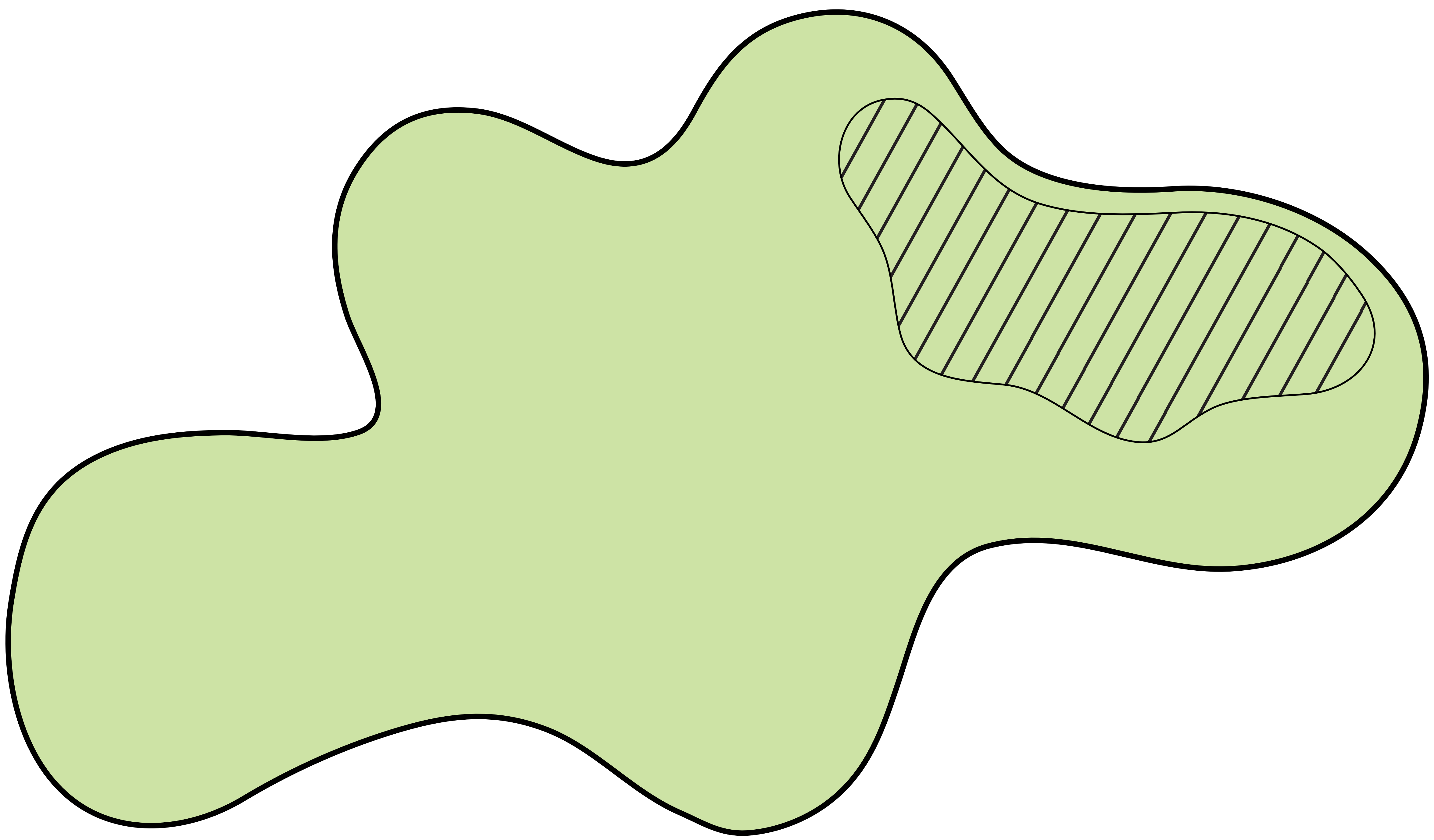
Posterior

$$p(u|\mathcal{Q}) \propto p(\mathcal{Q}|u) \times p(u)$$



Posterior

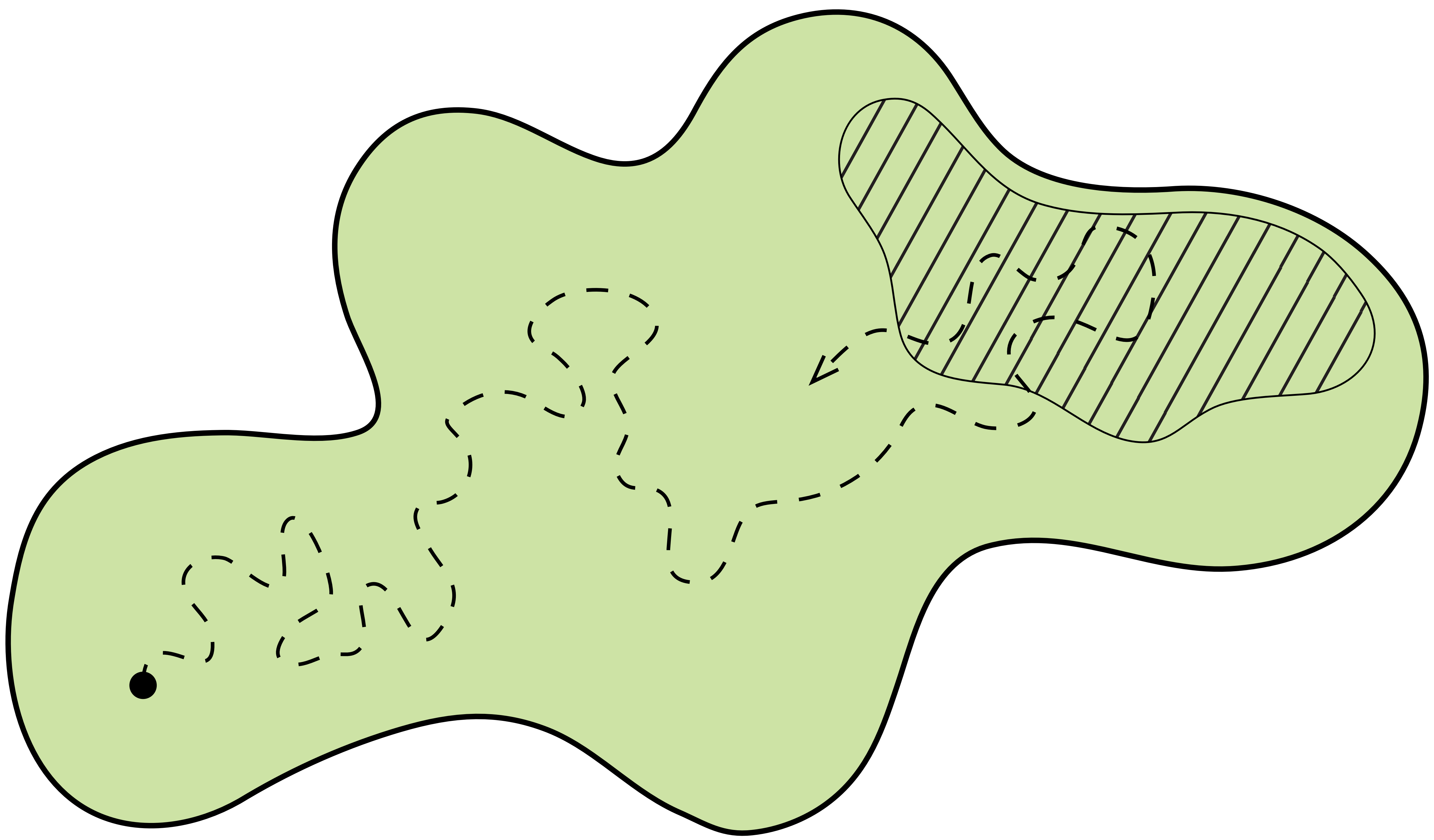
$$p(u|\mathcal{Q}) \propto p(\mathcal{Q}|u) \times p(u)$$



Monte Carlo

Posterior

$$p(u|\mathcal{Q}) \propto p(\mathcal{Q}|u) \times p(u)$$



Markov chain Monte Carlo

Conclusion

$$\mathcal{Q} \longrightarrow \square \longrightarrow u$$

Indirect	Primary
Incomplete	Comprehensive
Noisy	Efficient

Thank you! Questions?

Prior

$$p(u) = p(\boldsymbol{\theta})$$

$$\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$
$$\mu_u \sim \mathcal{N}(\mu_0, \sigma_0^2)$$
$$\sigma_u^2 \sim \text{Scale-inv-}\chi^2(\nu_u, \tau_u^2)$$
$$\sigma_\epsilon^2 \sim \text{Scale-inv-}\chi^2(\nu_\epsilon, \tau_\epsilon^2)$$

Correlations

$$u \sim \mathcal{GP}(\mu, k)$$

$$k(r, r') = \sigma_u^2(\eta\, k_{\mathrm{SE}}(r, r') + (1 - \eta)\, k_{\mathrm{OU}}(r, r'))$$

$$k_{\mathrm{OU}}(r, r') = \exp\left(-\frac{|\,\|r\| - \|r'\|\,|}{\ell_{\mathrm{OU}}}\right)$$

$$k_{\mathrm{SE}}(r, r') = \exp\left(-\frac{\|r - r'\|^2}{\ell_{\mathrm{SE}}^2}\right)$$

Likelihood

$$p(\mathcal{Q}|u) = p(\{q_i^{\mathrm{msr}}\}|\boldsymbol{\theta})$$

$$q = f(u) \qquad \mathbf{q}^{\mathrm{msr}} = \mathbf{q} + \boldsymbol{\epsilon}$$

$$\mathbf{q}^{\mathrm{msr}}|\boldsymbol{\theta} \sim \mathcal{N}(\mathbf{q}, \sigma_\epsilon^2 \mathbf{I})$$

Posterior

$$p(u|\mathcal{Q}) = p(\boldsymbol{\theta}|\{q_i^{\mathrm{msr}}\})$$

Metropolis-Hastings

$$\boldsymbol{\theta} \sim t_\nu \left(\hat{\boldsymbol{\theta}}, \alpha^2 \mathbf{J}^{-1}\right)$$