

Temperature-Centric Reliability Analysis and Optimization under Process Variation

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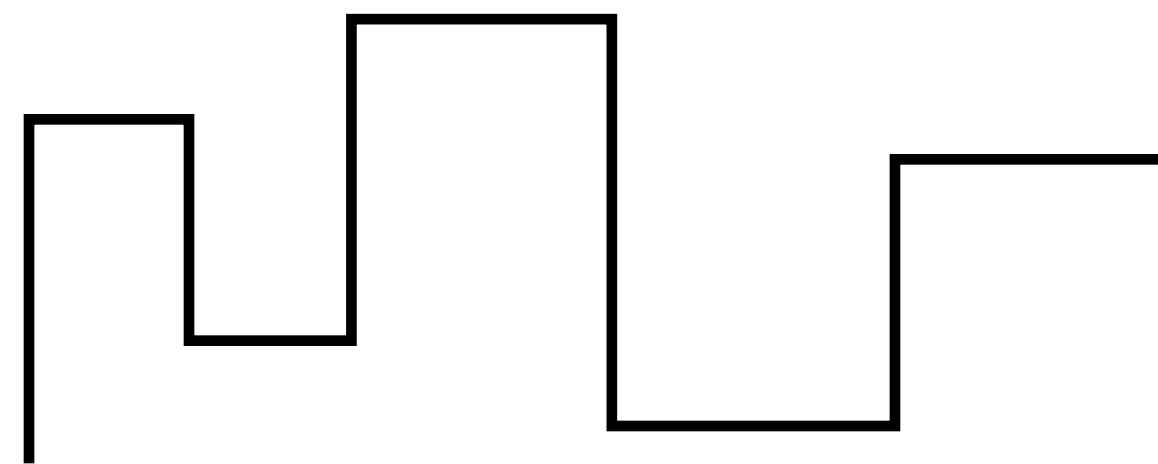
May 2015

Outline

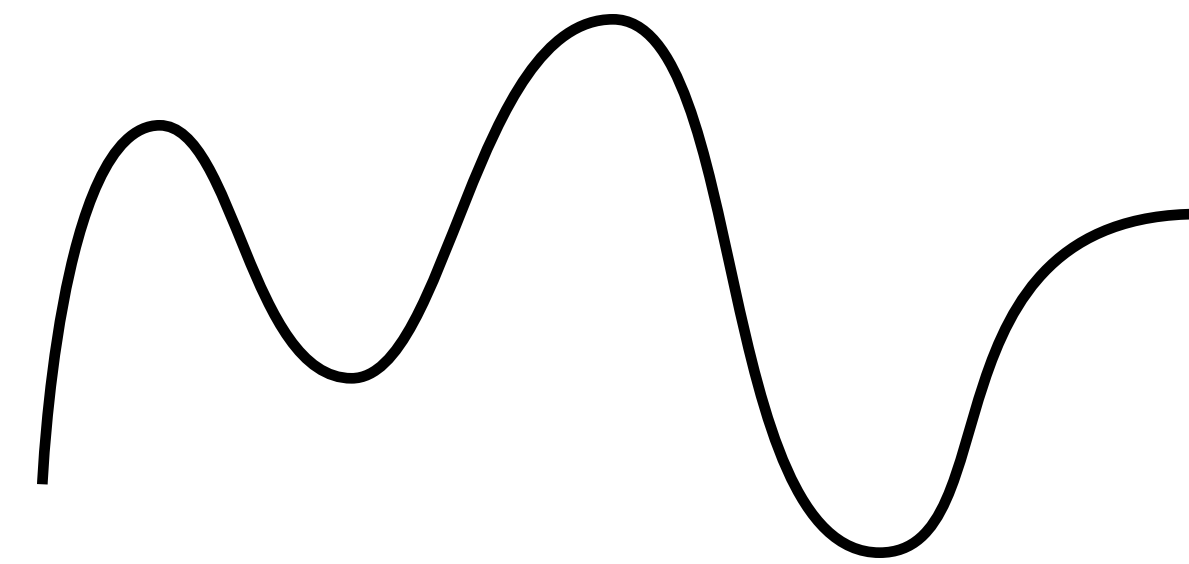
- * Temperature analysis
- * Process variation
- * Reliability analysis
- * Reliability optimization

Temperature Analysis

* Transient



Power



Temperature

Temperature Analysis

* Static steady state

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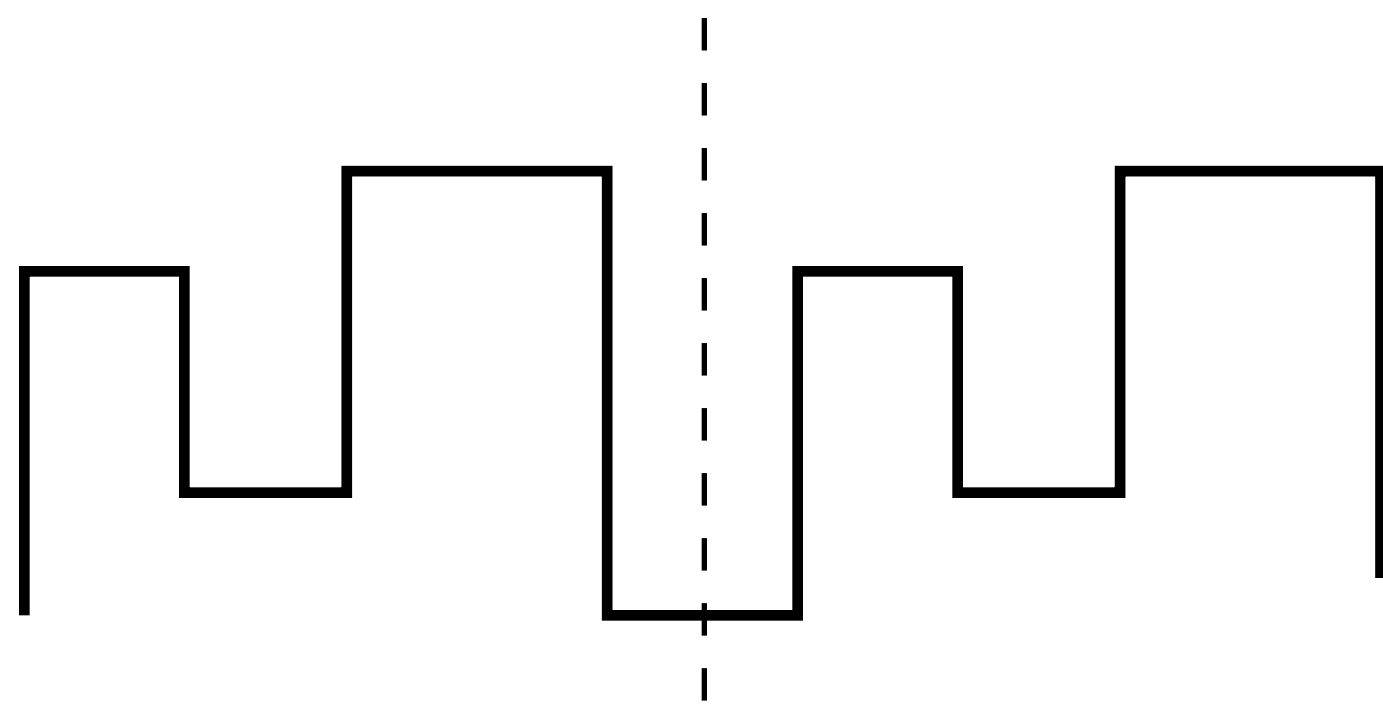
Power

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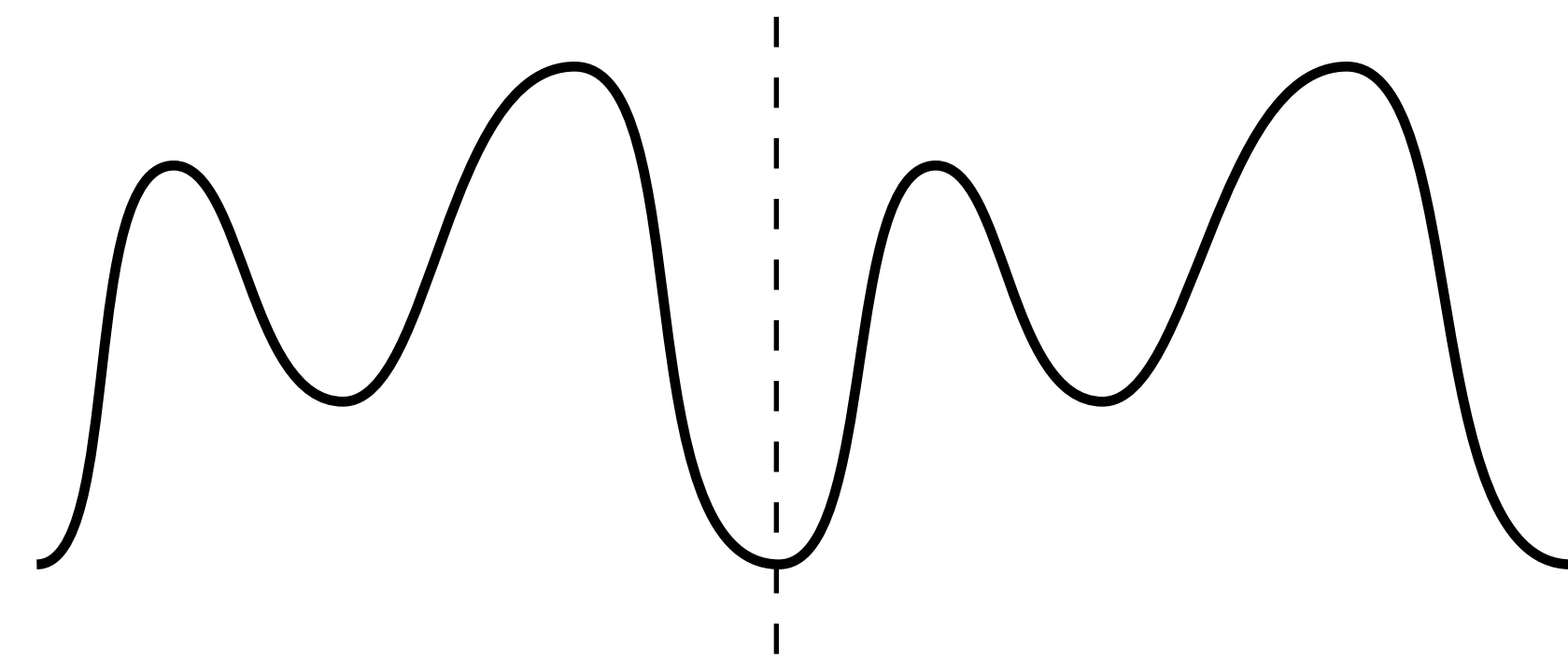
Temperature

Temperature Analysis

* Dynamic steady state

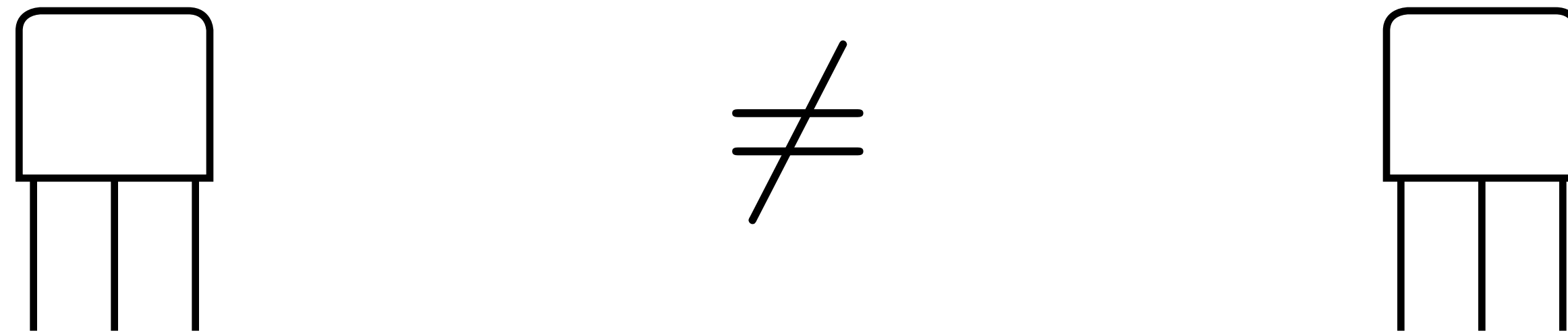


Power

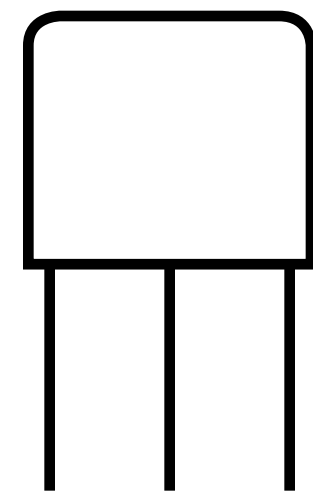


Temperature

Process Variation



Process Variation

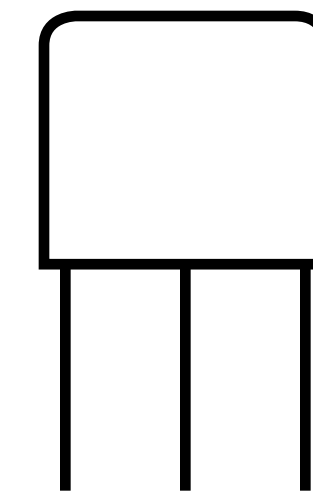


$$L_{\text{eff}} \neq L_{\text{eff}}$$

$$T_{\text{ox}} \neq T_{\text{ox}}$$

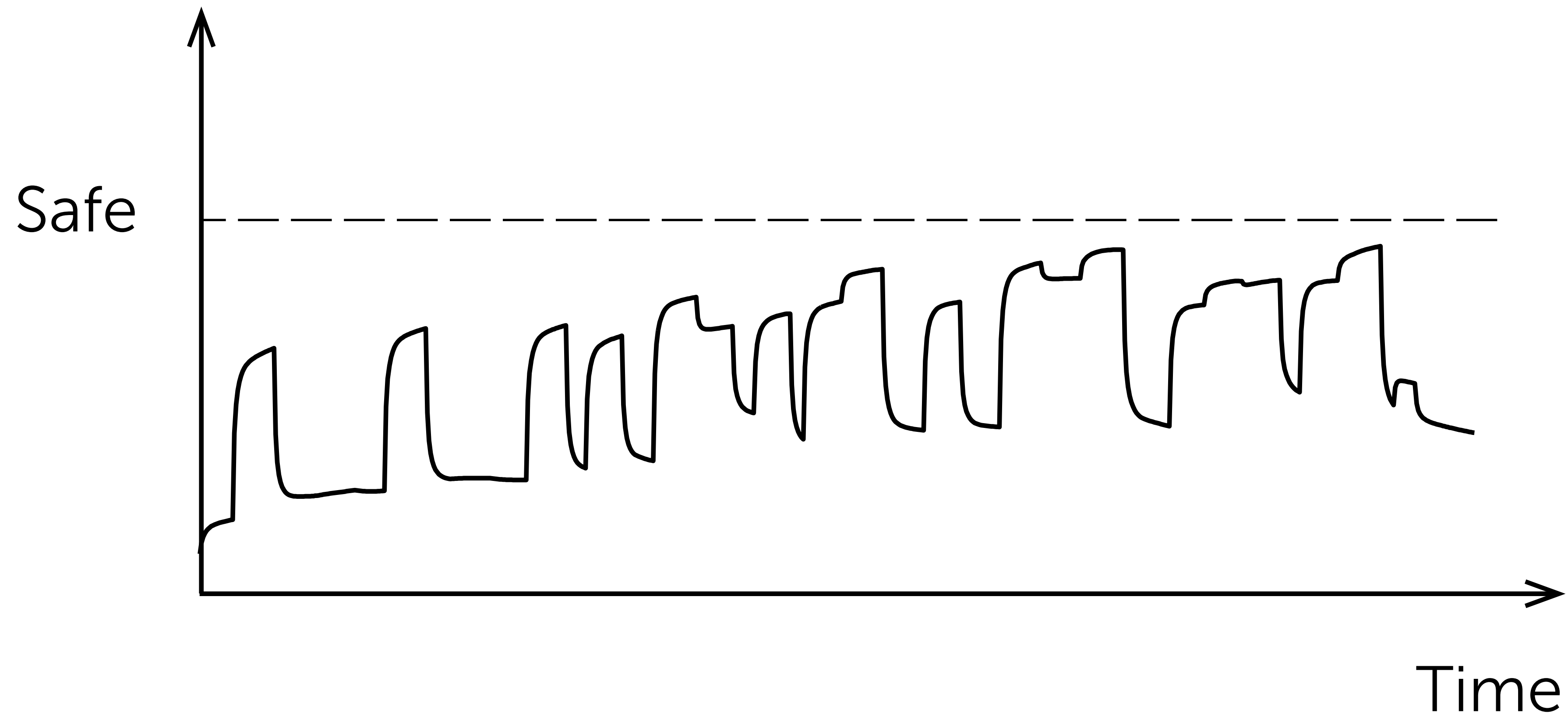
$$V_{\text{th}} \neq V_{\text{th}}$$

...



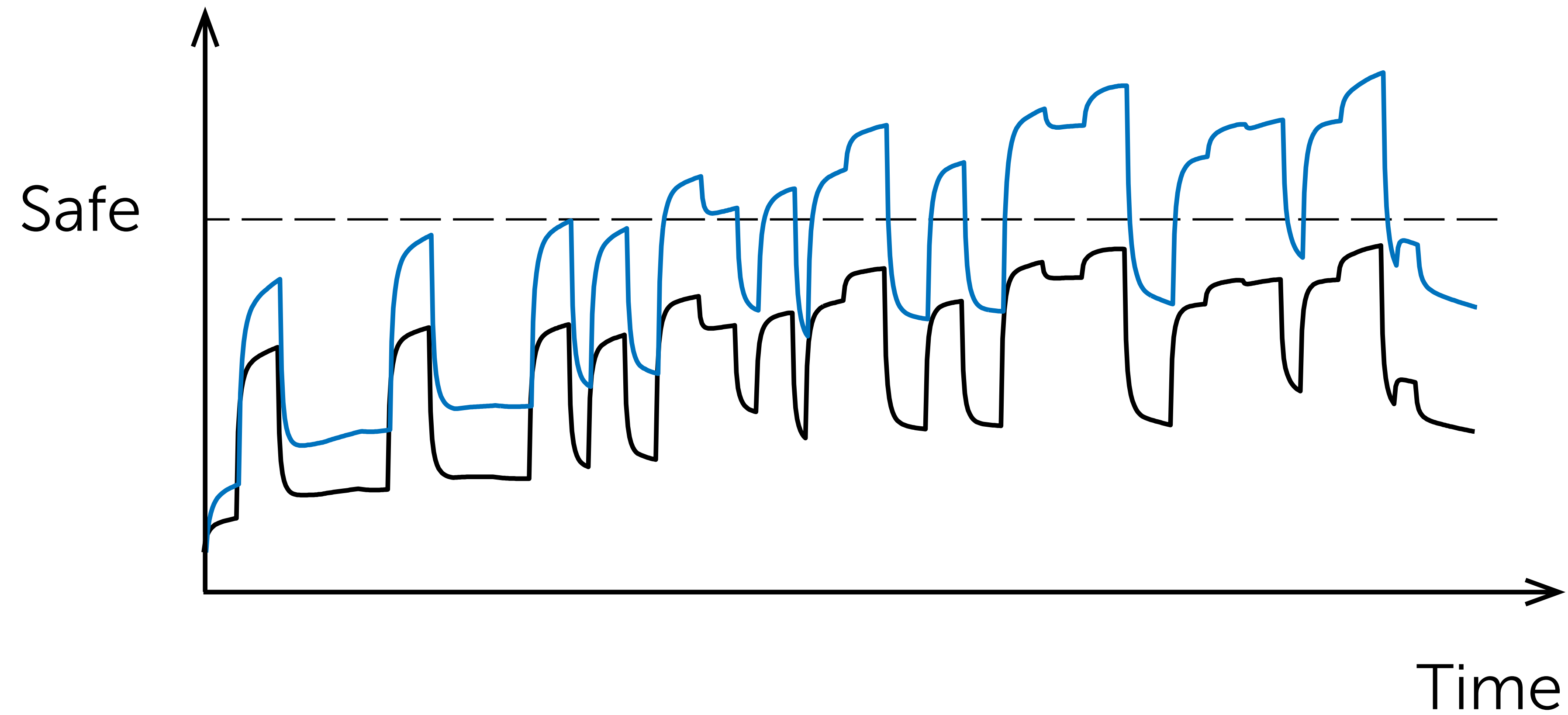
Process Variation

Temperature

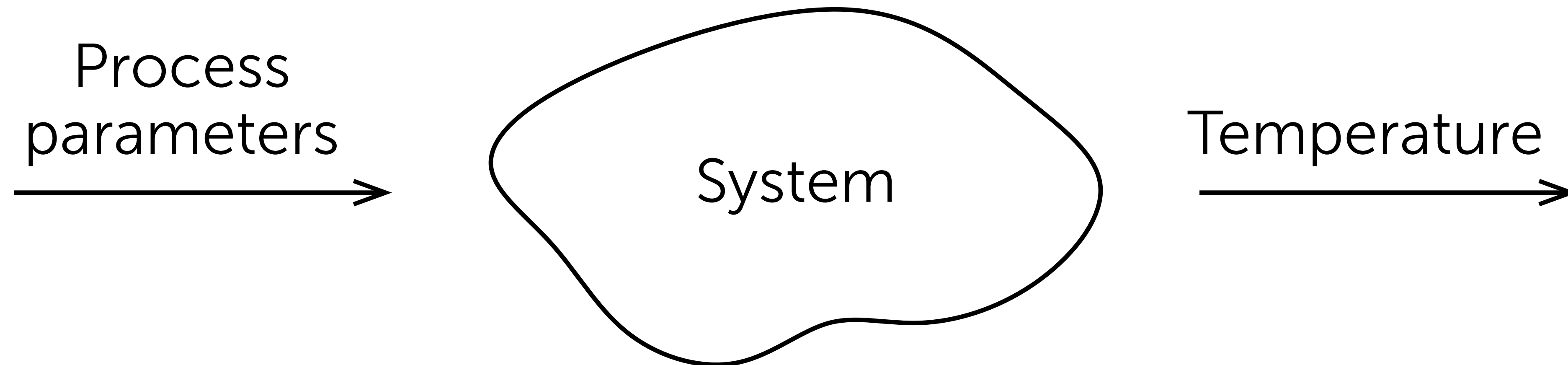


Process Variation

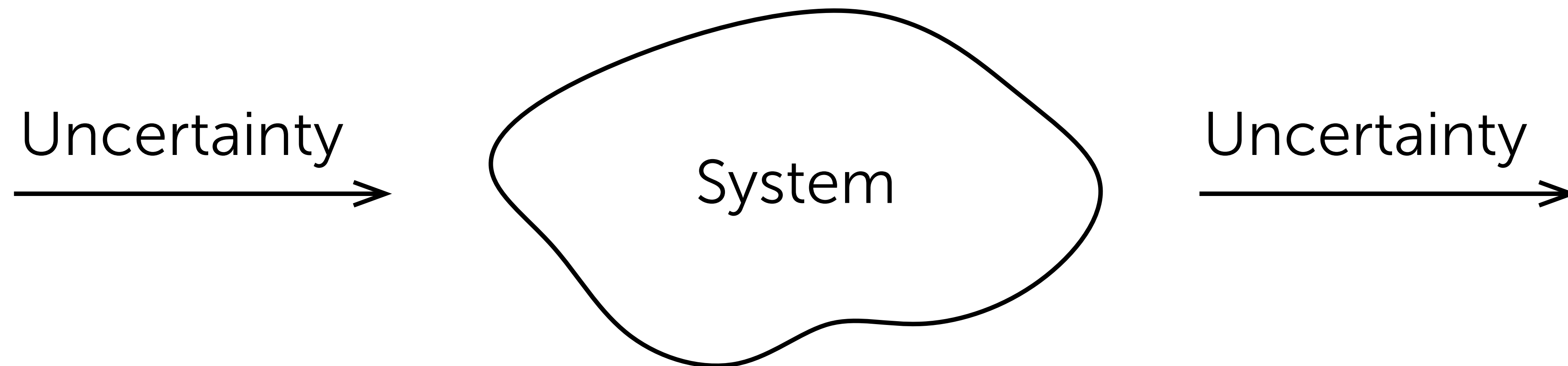
Temperature



Electronic System

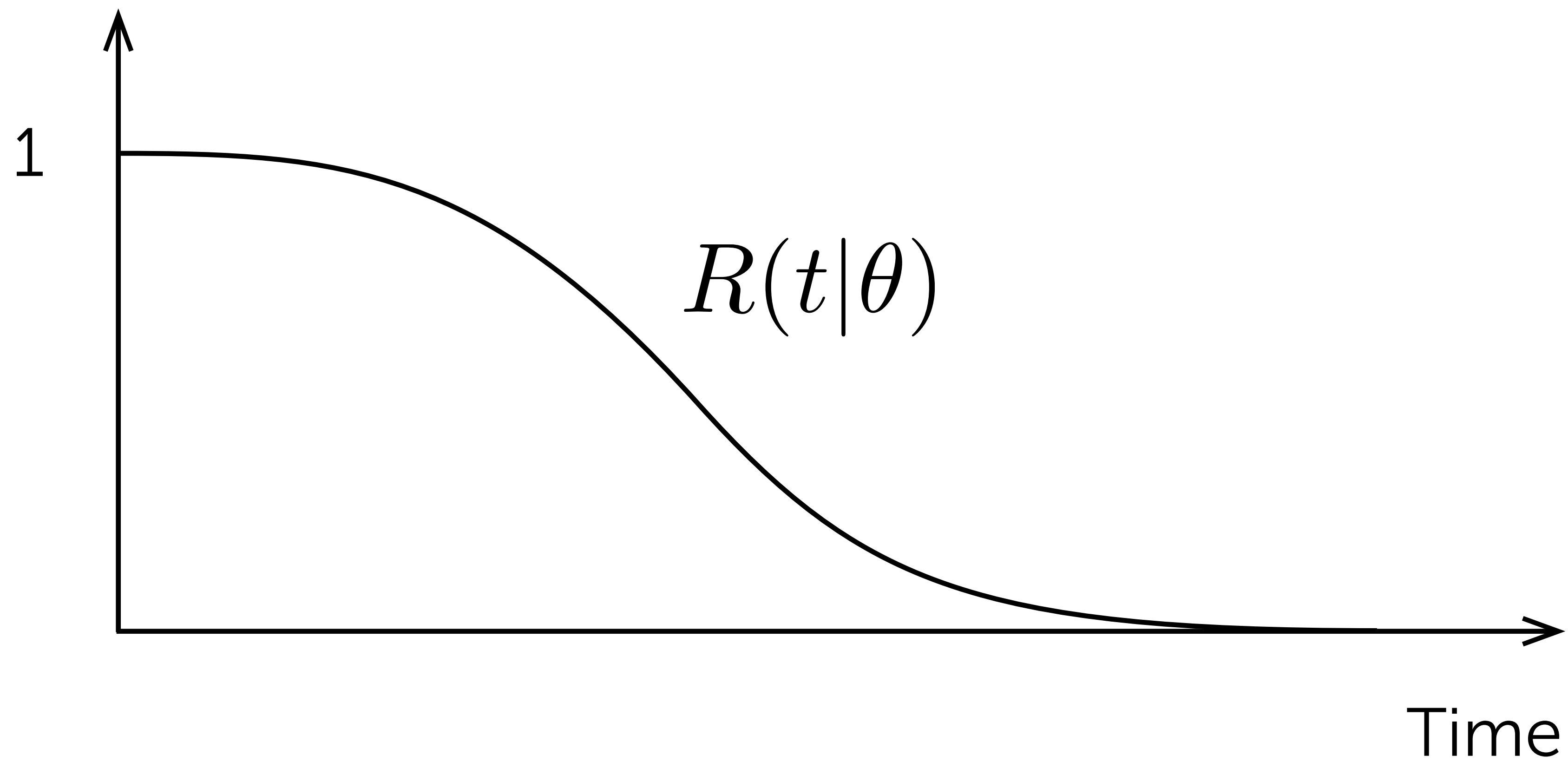


Electronic System

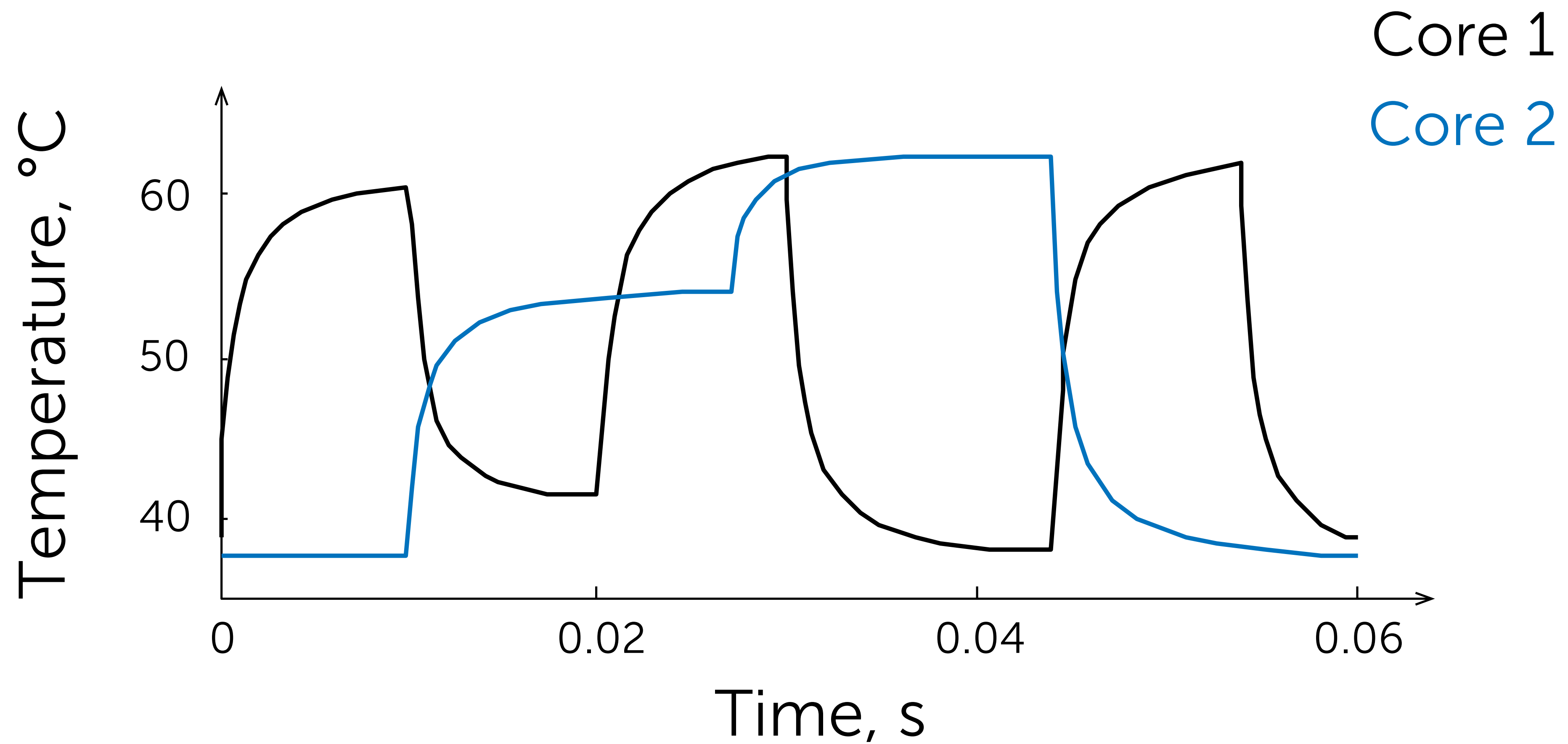


Reliability Analysis

Survival function



Thermal-Cyclic Fatigue



Thermal-Cyclic Fatigue

$$N = \alpha(\Delta T)^\beta \exp \left\{ \frac{\gamma}{T_{\max}} \right\}$$

Goal

Given:

- * Electronic system
- * Process variation

Perform:

- * Reliability analysis

Such that:

- * Accurate and computationally efficient

Reliability Analysis

$$R(t|\theta)$$

Reliability Analysis

$$R(t|\theta)$$

Reliability Analysis

$$\theta = \{\theta_i\}_{i=1}^n$$

Reliability Analysis

$$\theta = \{\theta_i\}_{i=1}^n$$

$$\theta = \{\theta_i(\text{system})\}_{i=1}^n$$

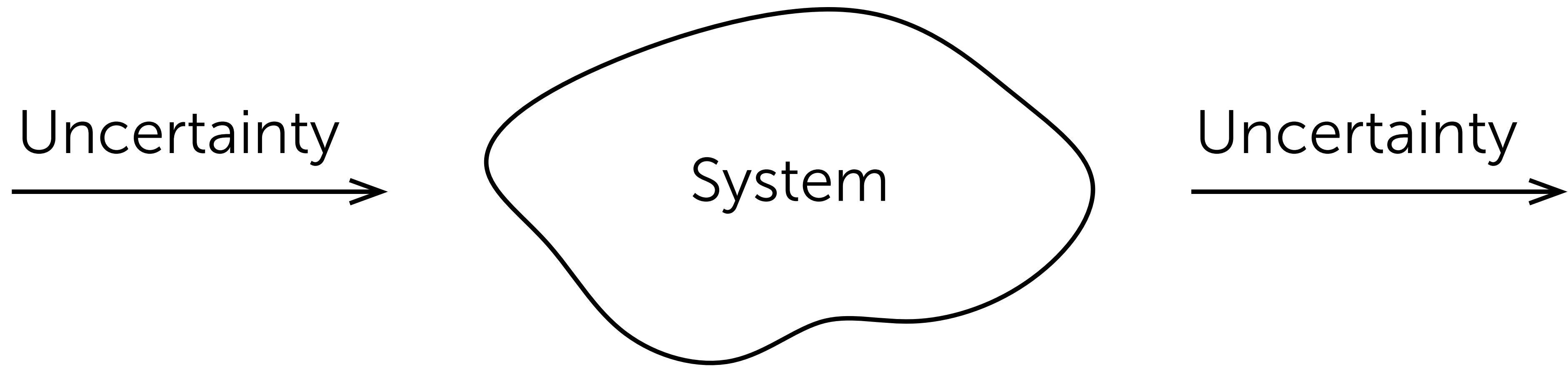
Reliability Analysis

$$\theta = \{\theta_i\}_{i=1}^n$$

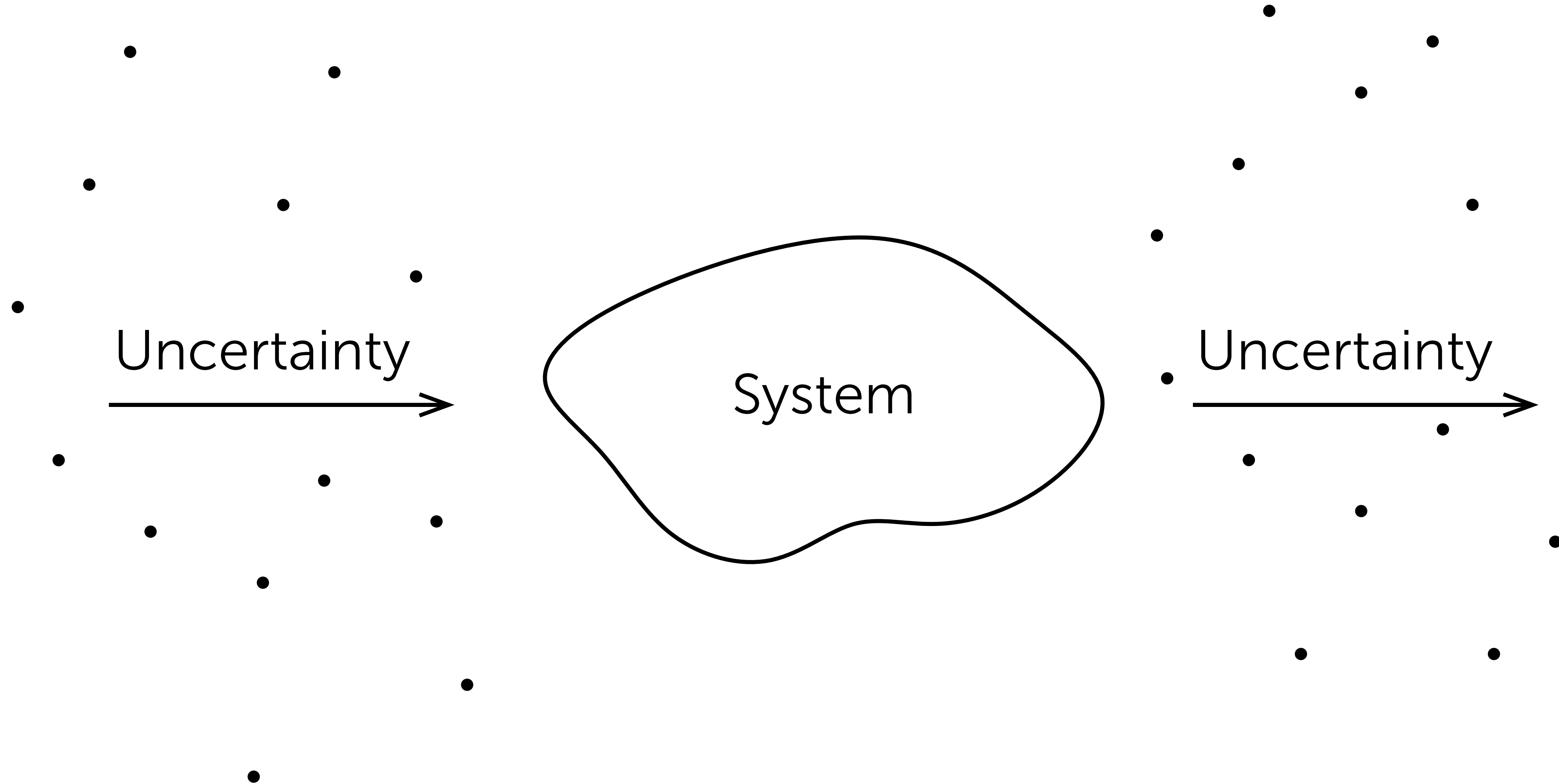
$$\theta = \{\theta_i(\text{system})\}_{i=1}^n$$

$$\theta = \{\theta_i(\text{system}(\text{uncertainty}))\}_{i=1}^n$$

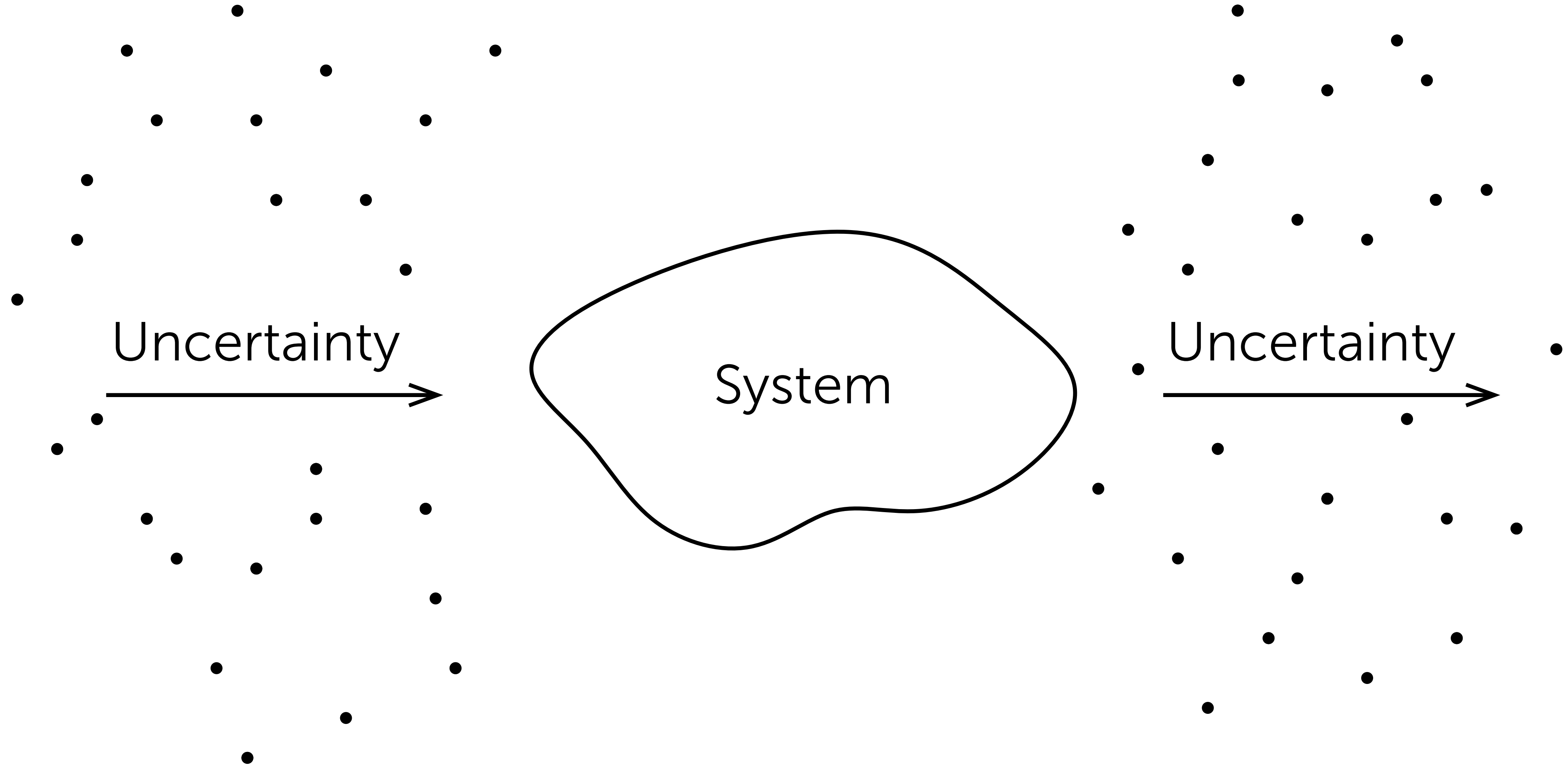
Uncertainty Quantification



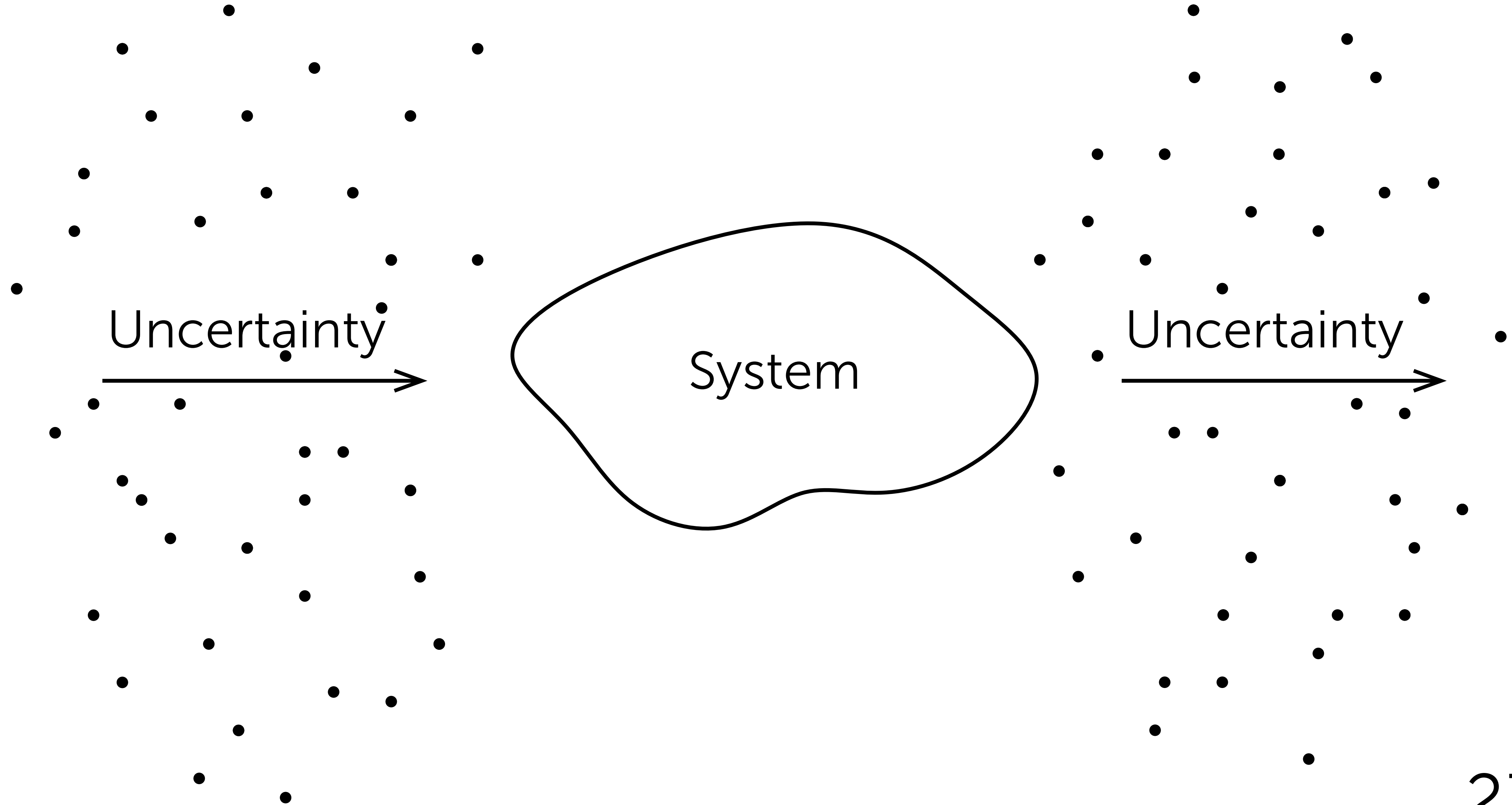
Monte Carlo



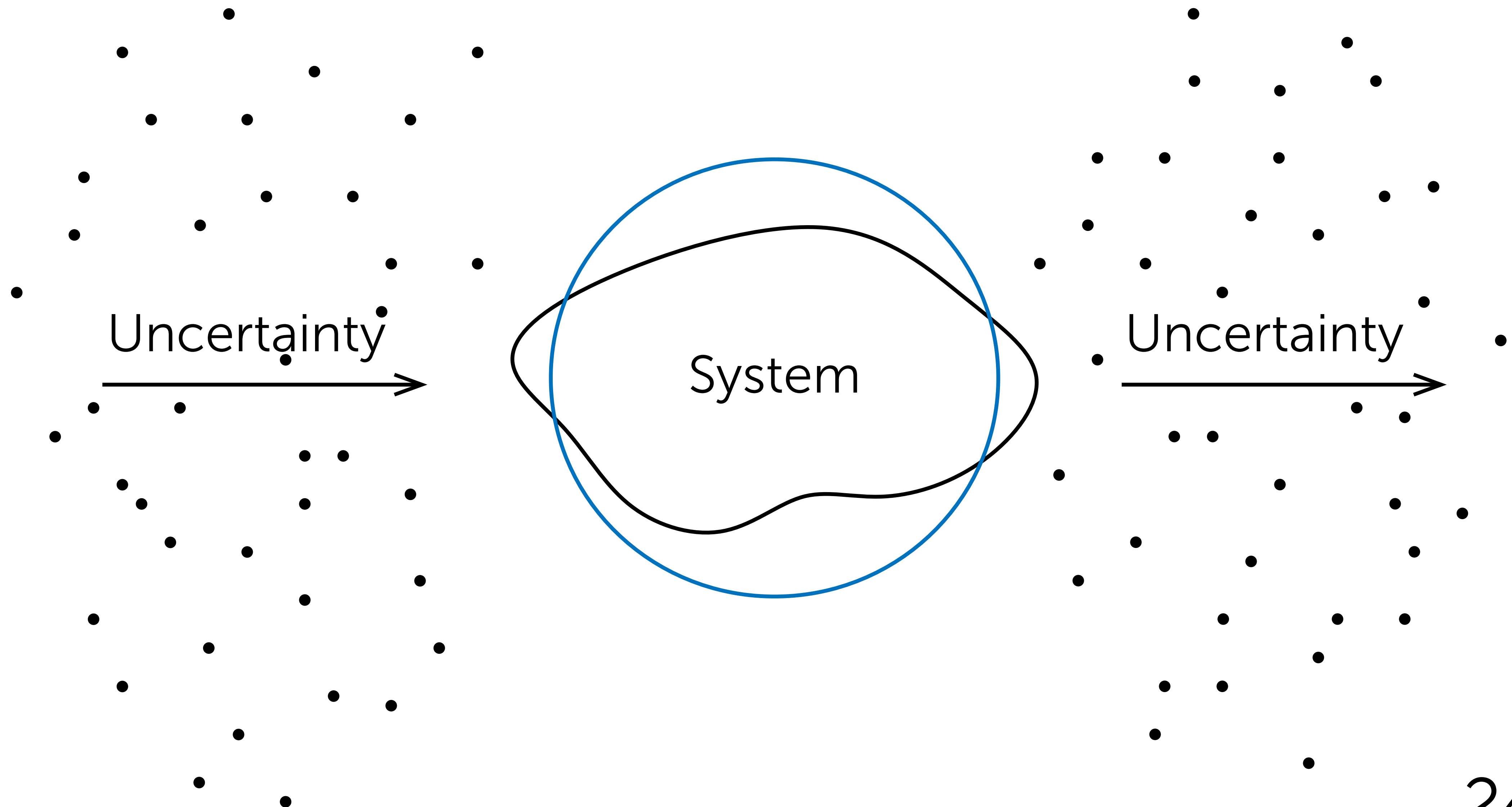
Monte Carlo



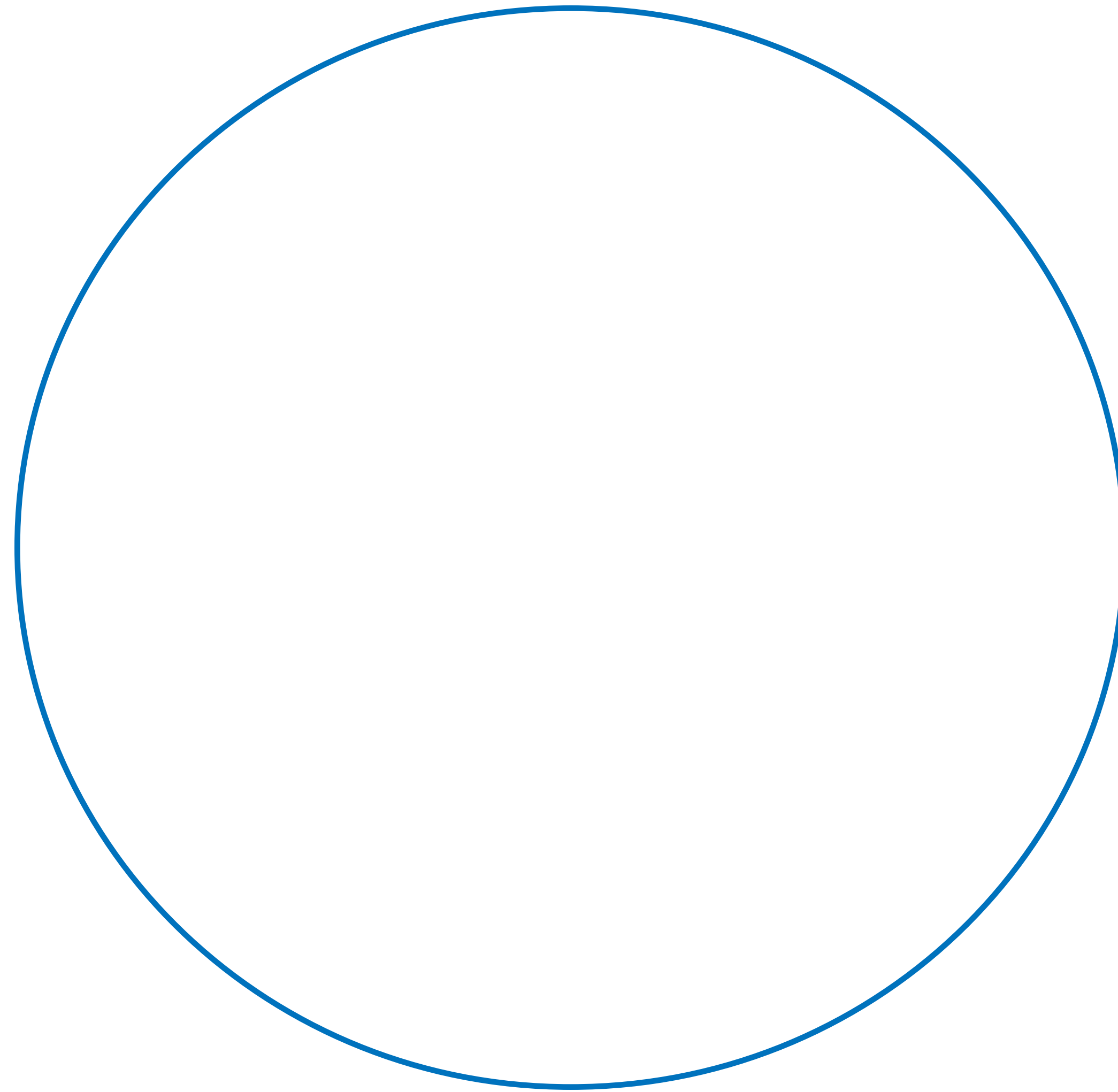
Monte Carlo



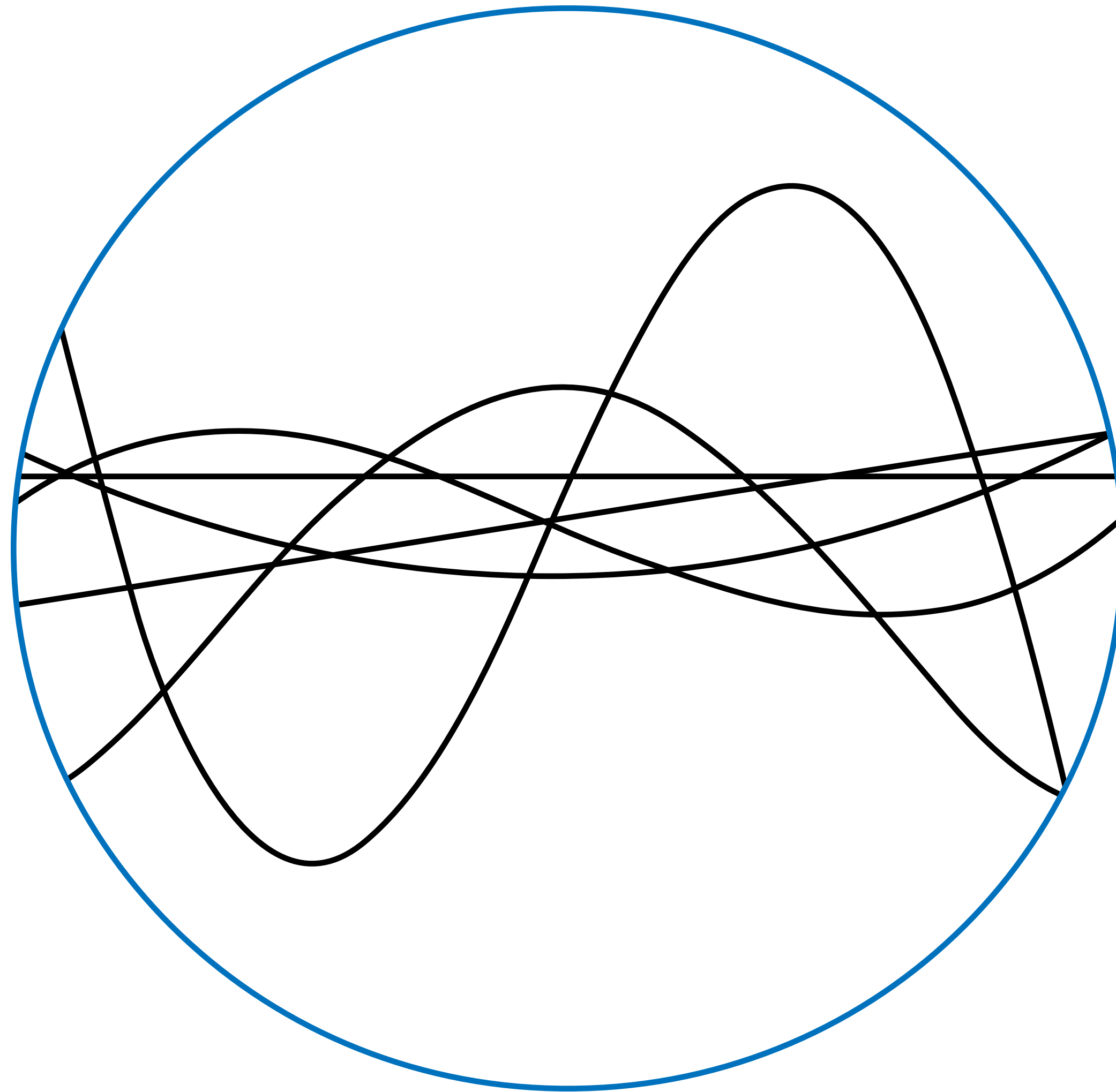
Solution



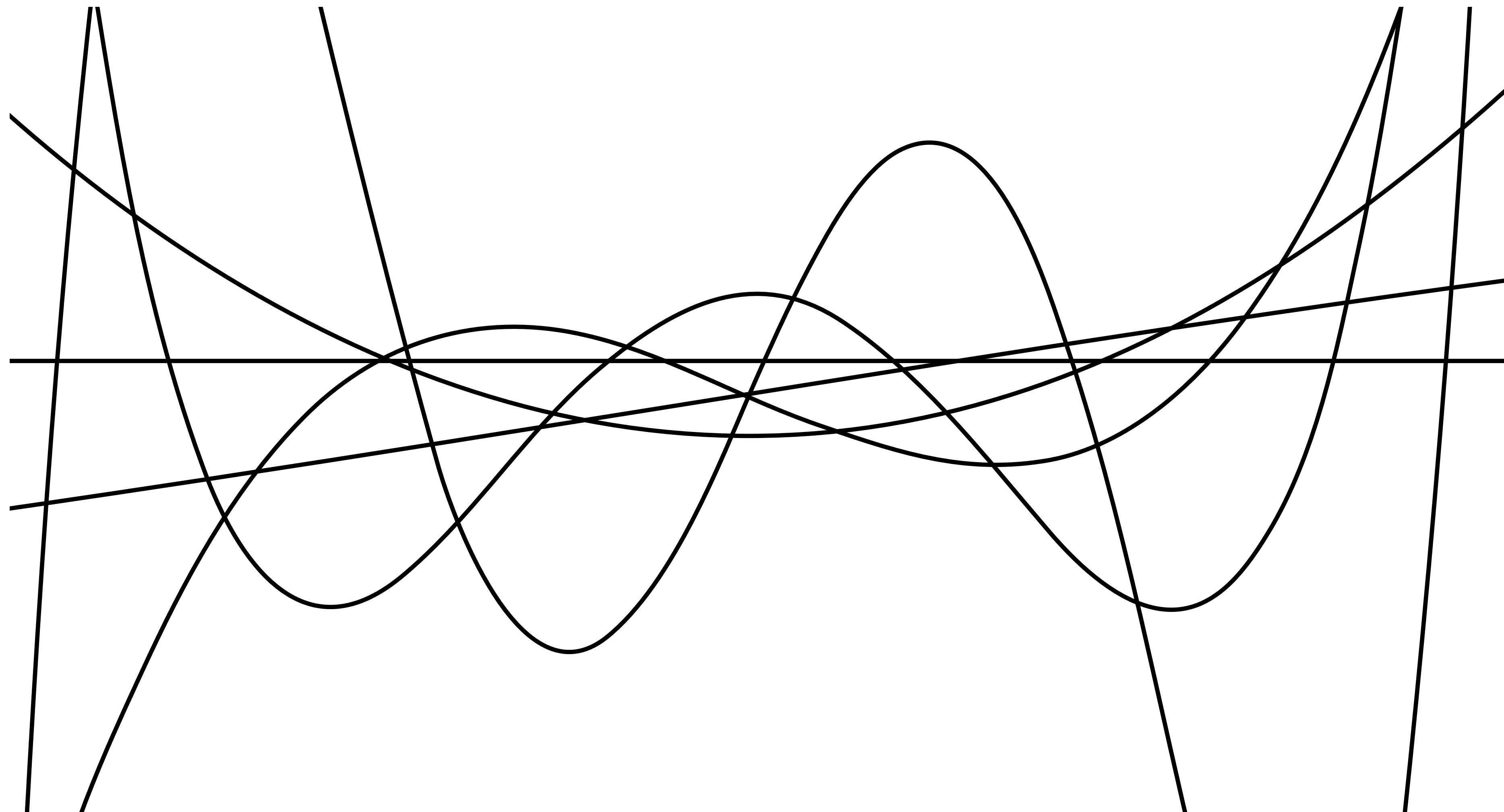
Polynomial Chaos



Polynomial Chaos



Polynomial Chaos



Polynomial Chaos

$$\theta_i(\xi) \approx \sum_j \hat{\theta}_{ij} \psi_j(\xi)$$

Orthogonal Polynomials

* Hermite	→	Gaussian
* Laguerre	→	Gamma
* Jacobi	→	Beta
* Gram–Schmidt	→	Arbitrary

Spectral Projection

$$\hat{\theta}_{ij} = \langle \theta_i, \psi_j \rangle = \int_{\Omega} \theta_i(\xi) \psi_j(\xi) f(\xi) d\xi$$

Numerical Integration

$$\int_D g(x) f(x) dx \approx \sum_i g(x_i) w_i$$

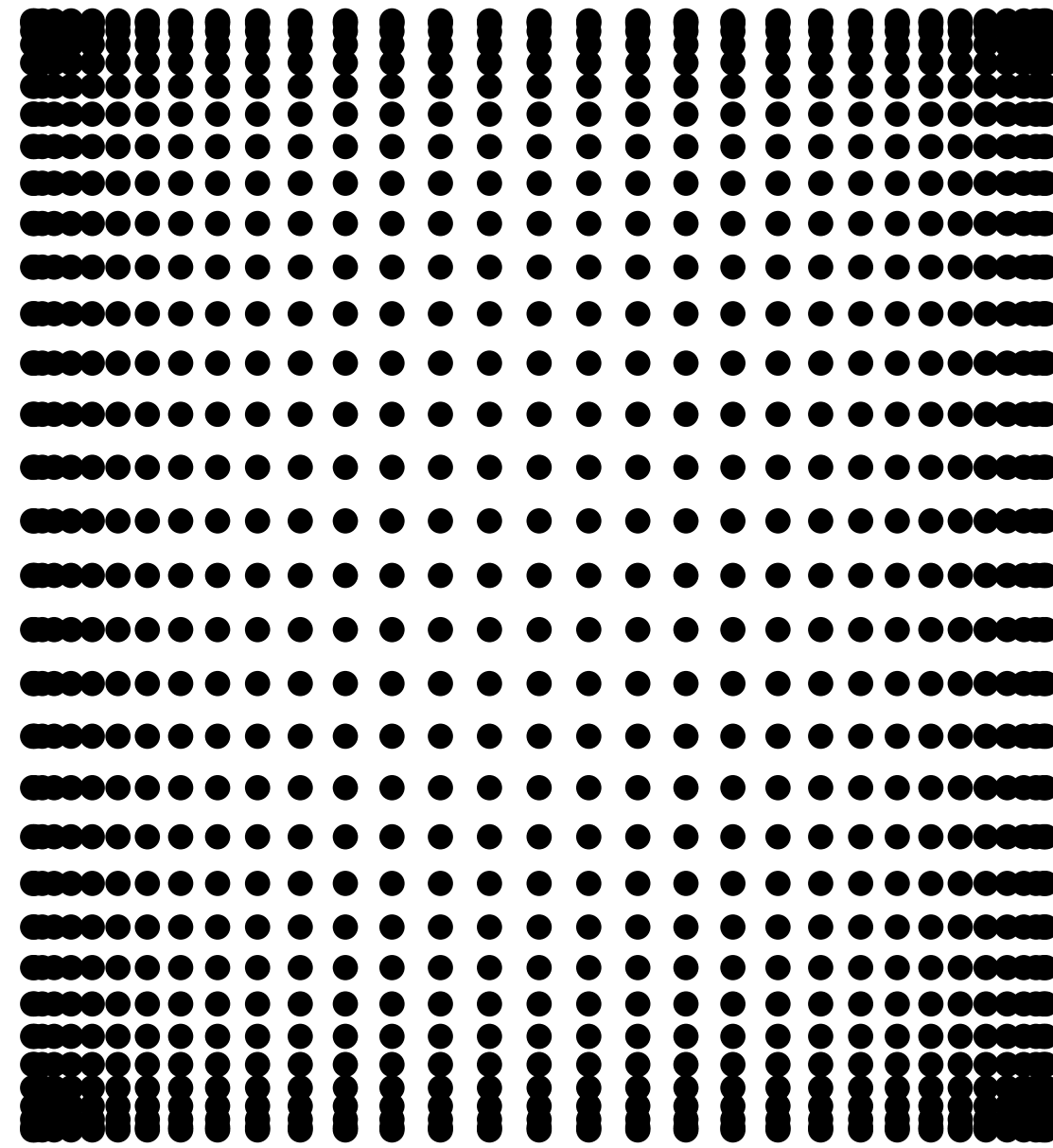
Quadrature Rules

- * Gauß–Hermite → Hermite
- * Gauß–Laguerre → Laguerre
- * Gauß–Jacobi → Jacobi
- * Golub–Welsch → Arbitrary

Quadrature Rules

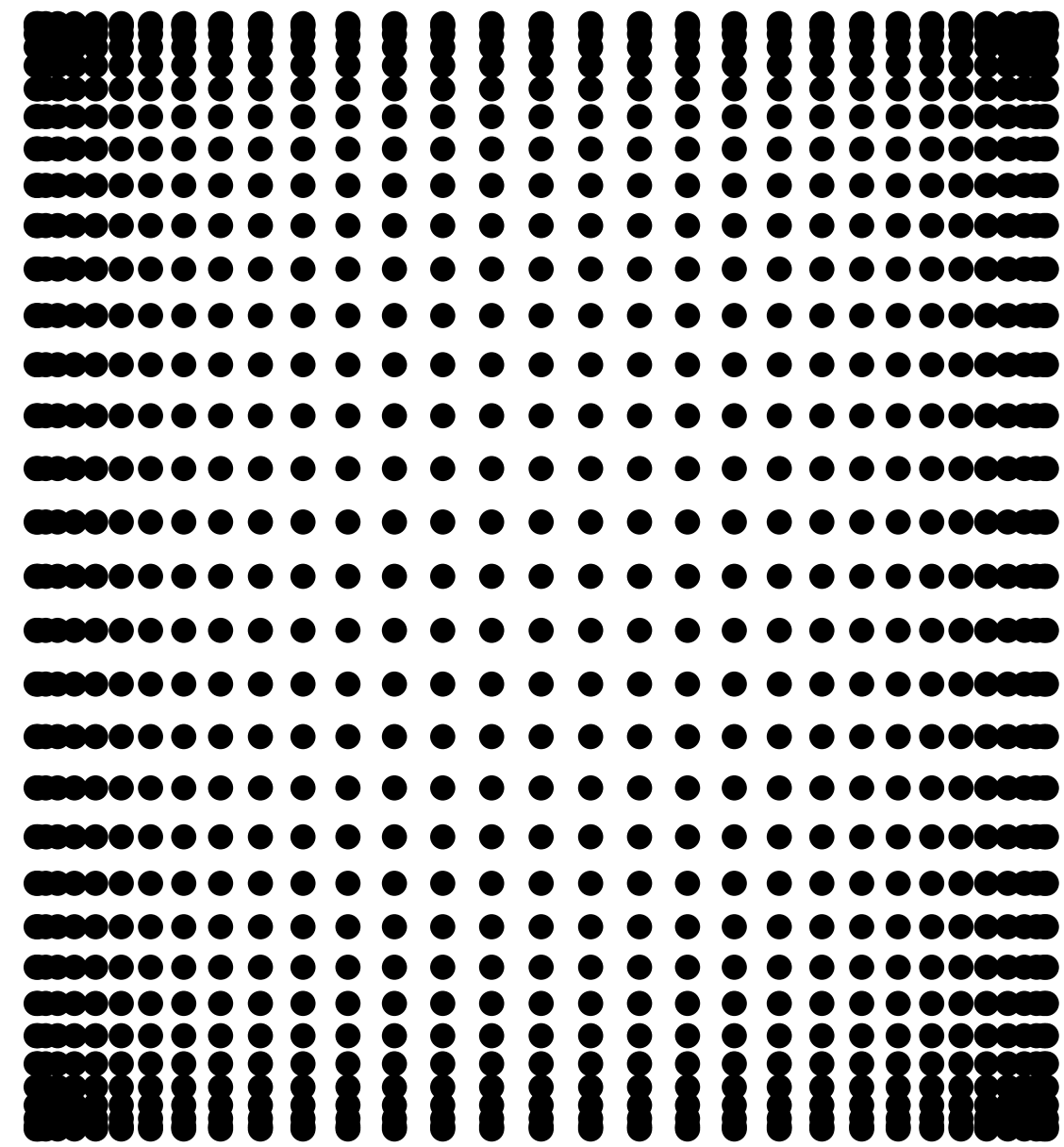
Abscissa	Weight
0.00000	0.94531
± 0.95857	0.39362
± 2.02018	0.01995

Multiple Dimensions

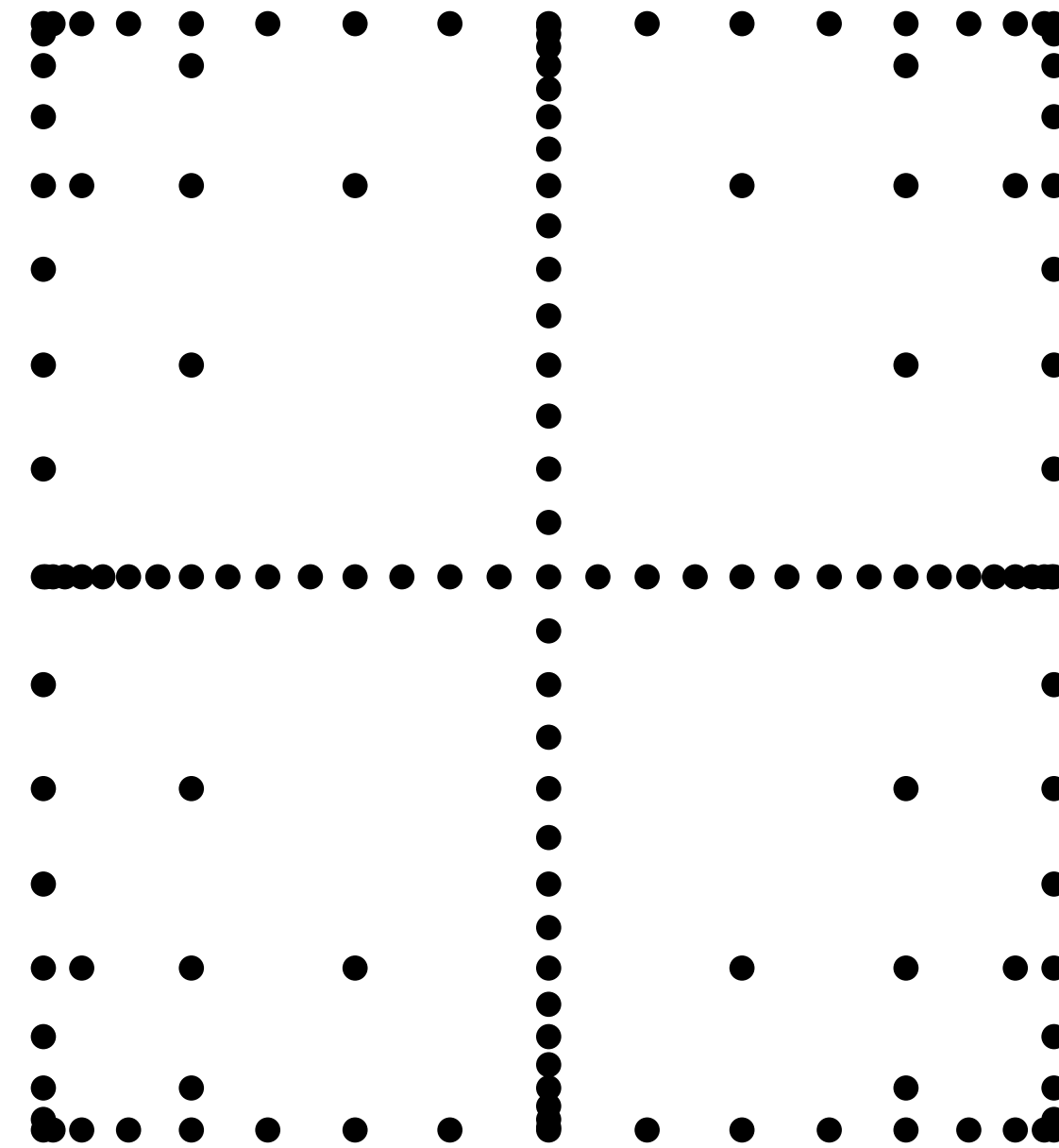


Full-tensor
product

Multiple Dimensions

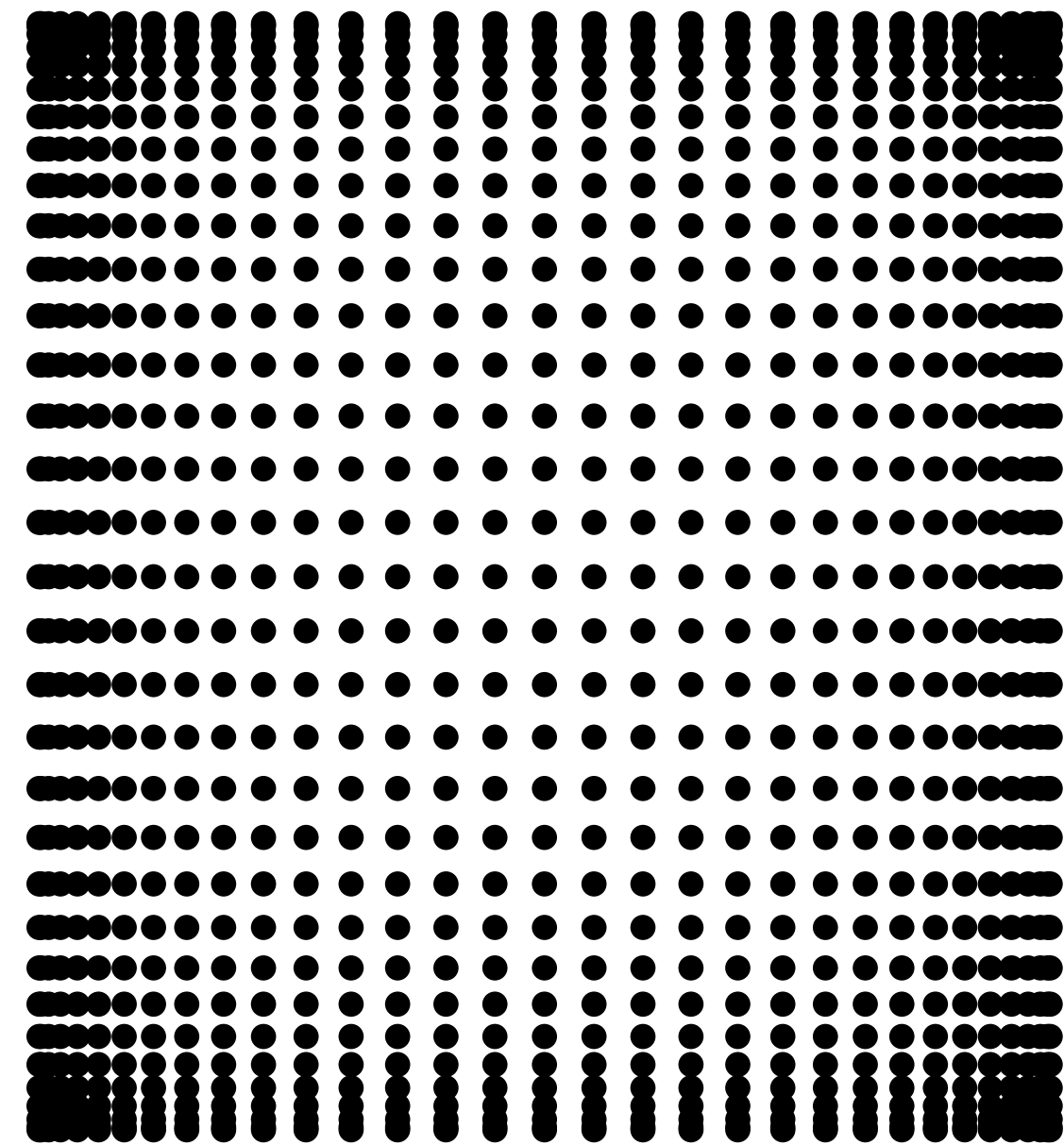


Full-tensor
product

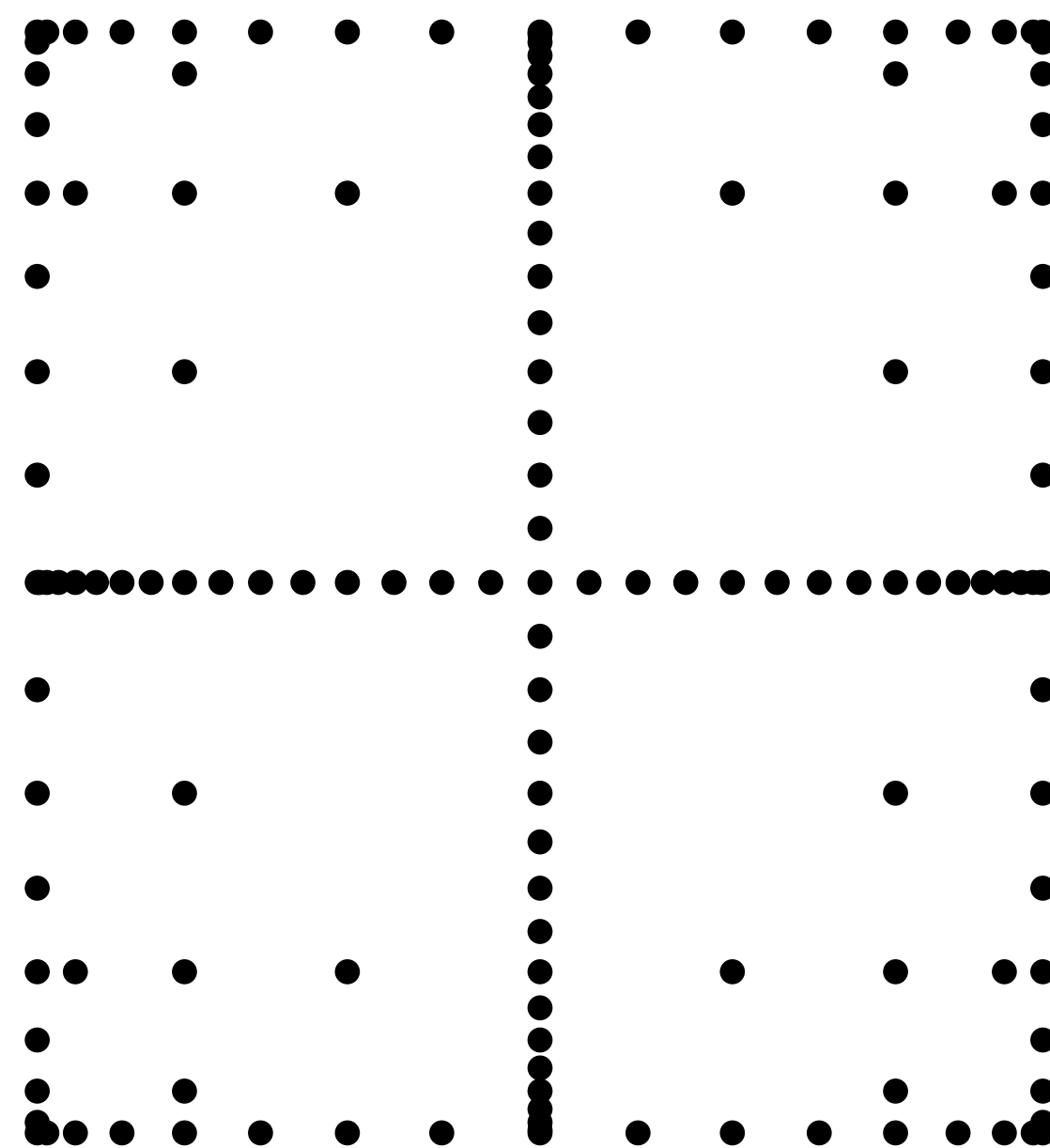


Isotropic
sparse grid

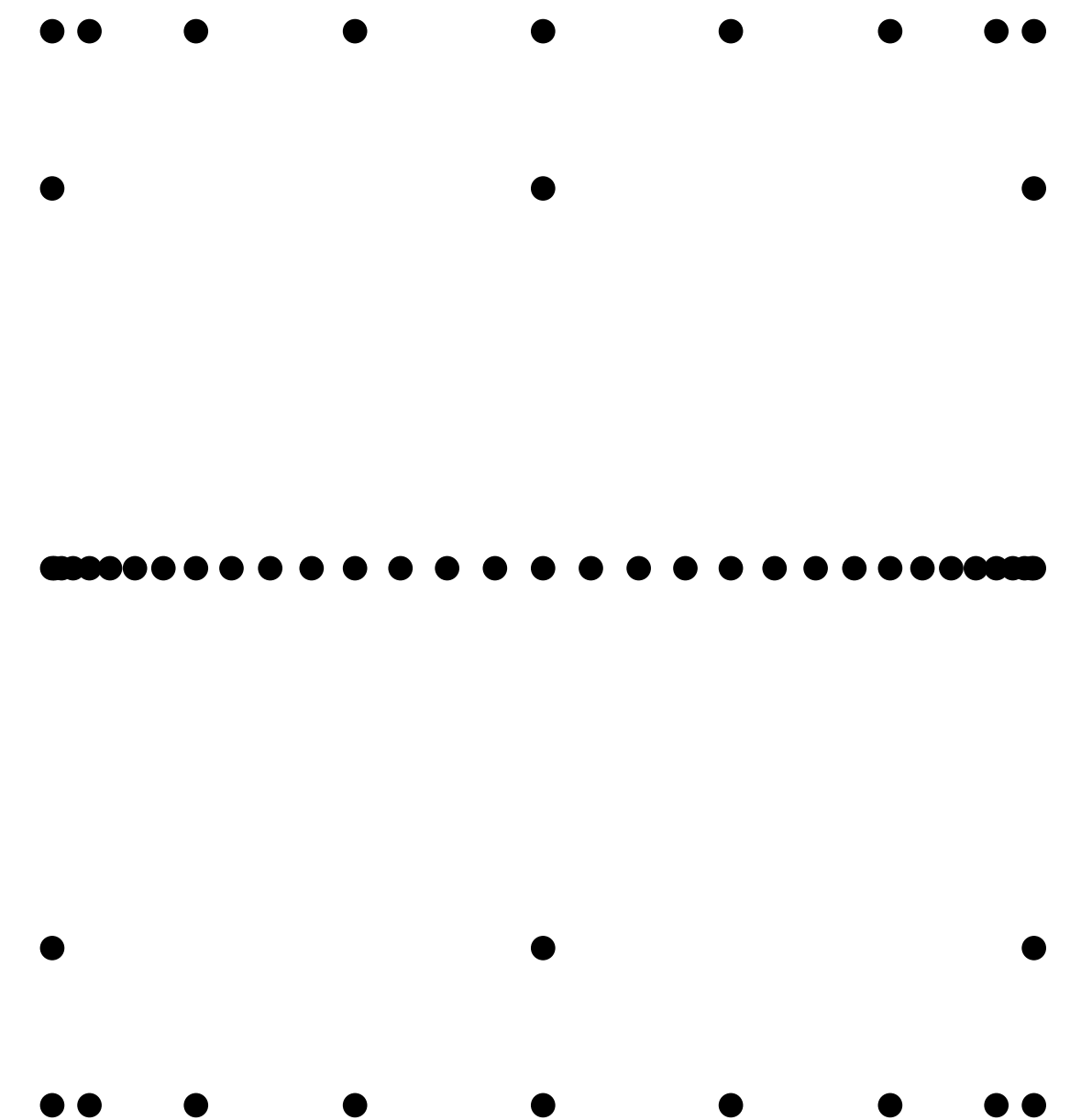
Multiple Dimensions



Full-tensor
product grid

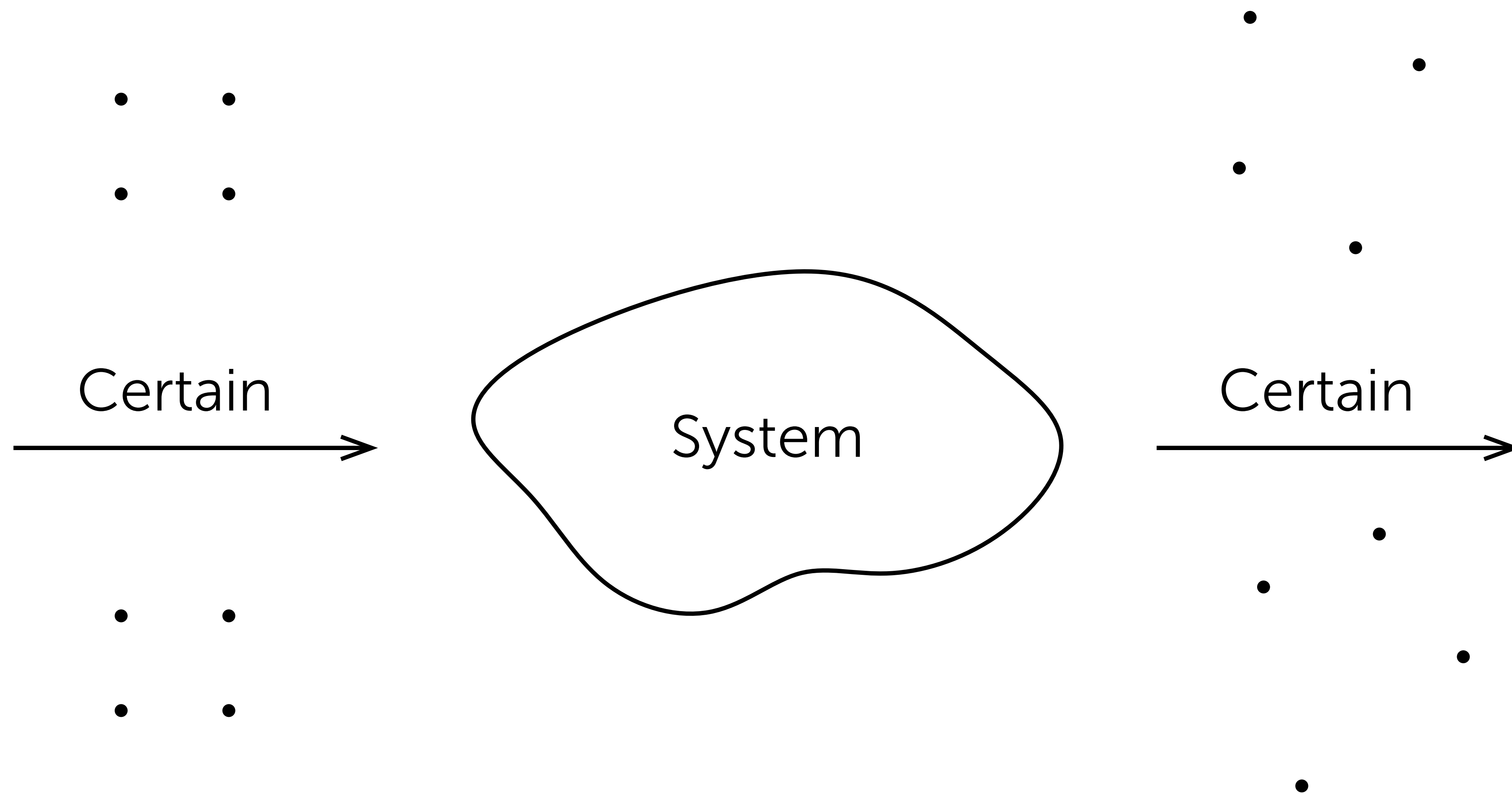


Isotropic
sparse grid

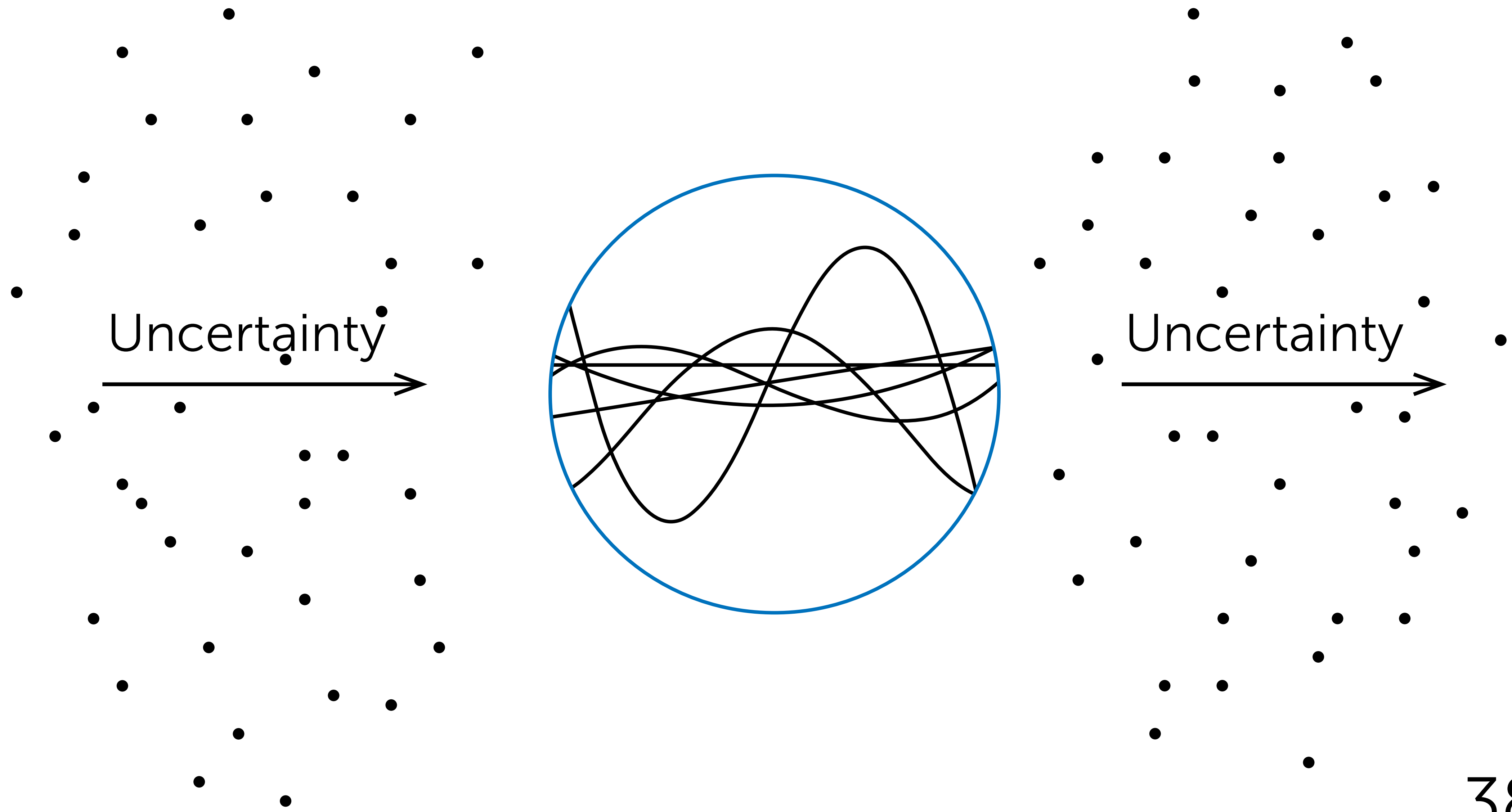


Anisotropic
sparse grid

Spectral Projection



Post-Processing



Post-Processing

$$\theta_i(\xi) \approx \sum_j \hat{\theta}_{ij} \psi_j(\xi)$$

$$\mathbb{E}[\theta_i] \approx \hat{\theta}_{i0}$$

$$\text{Var}[\theta_i] \approx \sum_{j>0} \hat{\theta}_{ij}^2$$

Reliability Analysis

$$R(t|\theta)$$

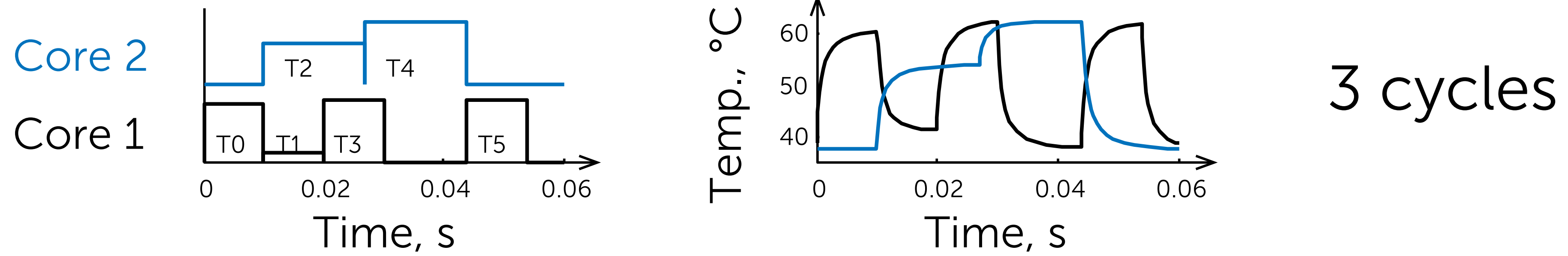
Reliability Analysis

$$R(t|\theta)$$

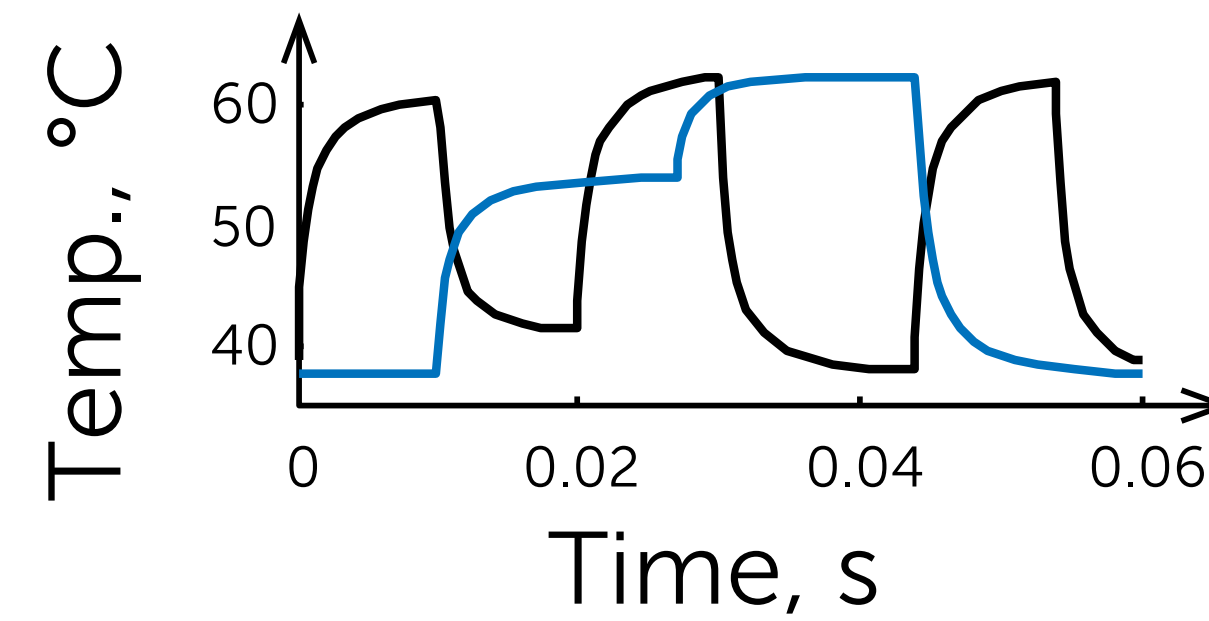
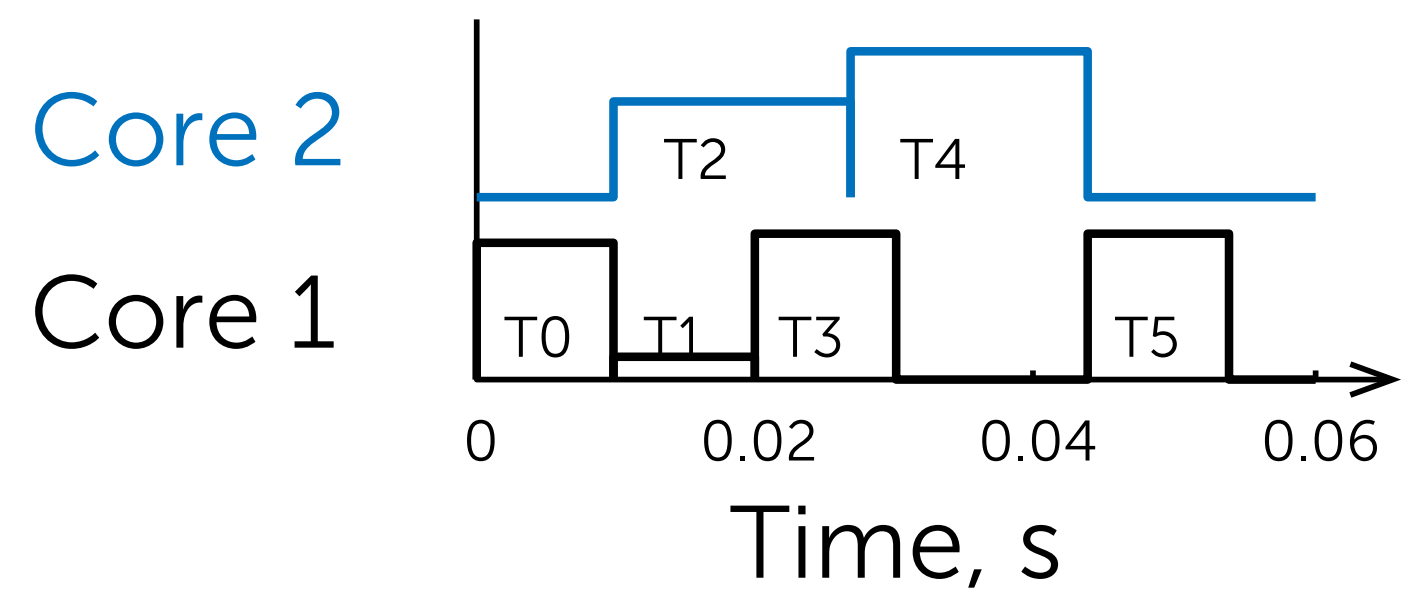
Reliability Optimization

- * Multiprocessor platform
- * Process variation
- * Periodic application
- * Thermal-cyclic fatigue

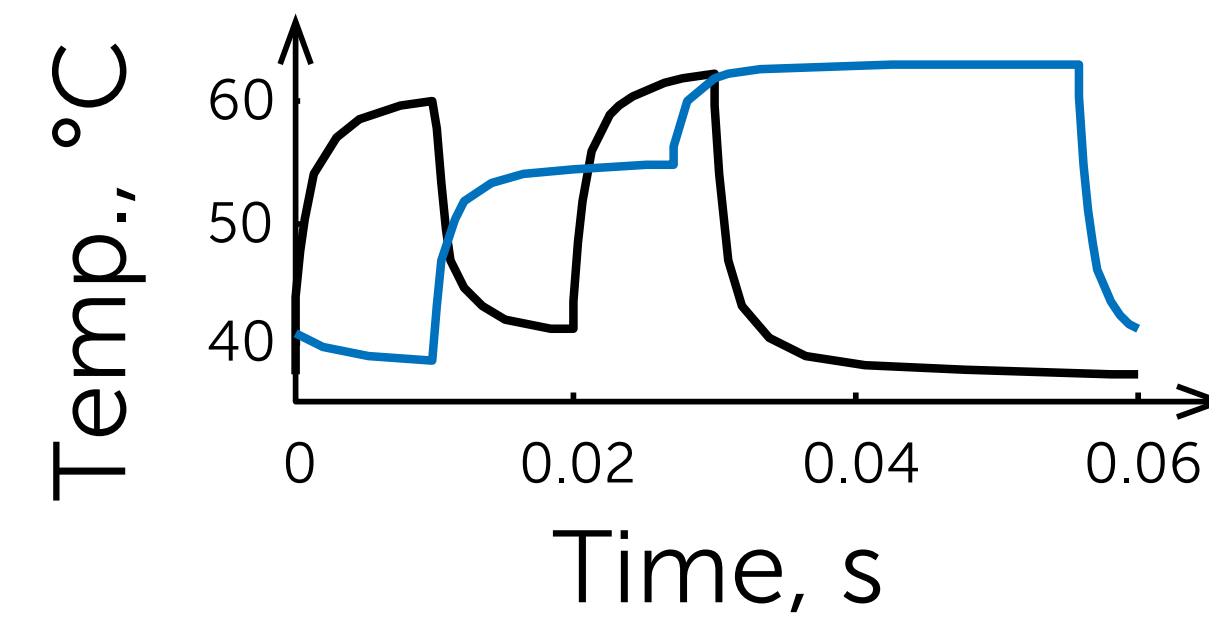
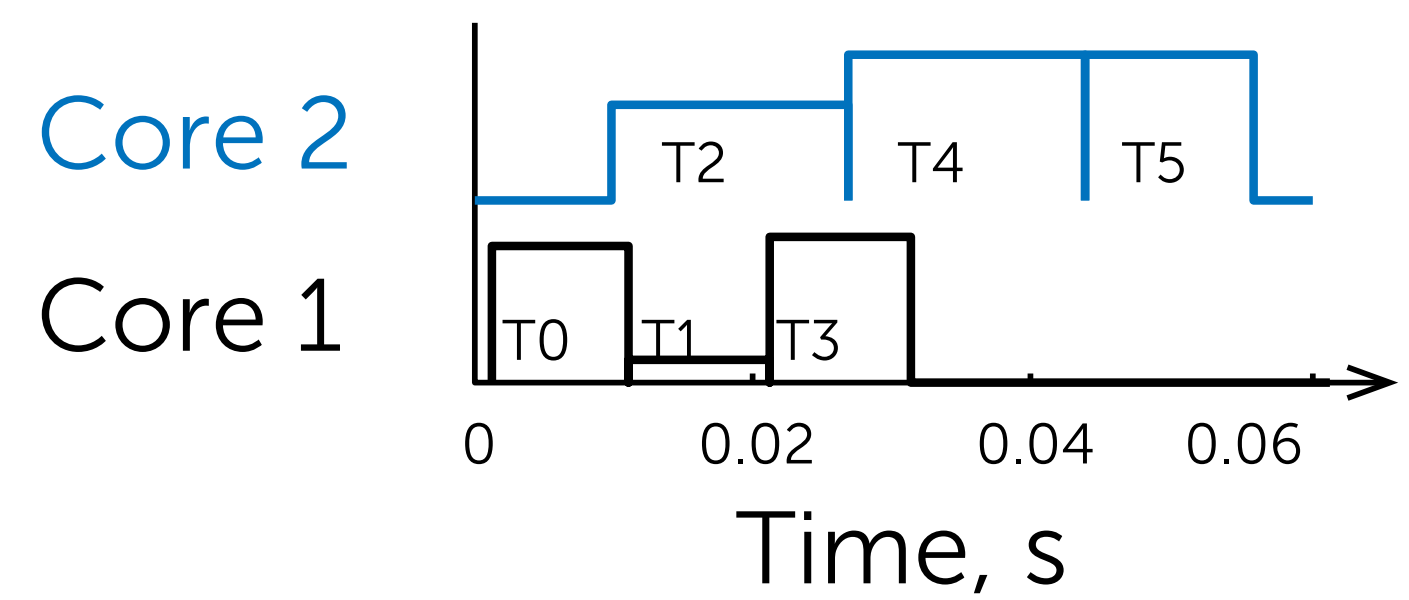
Reliability Optimization



Reliability Optimization

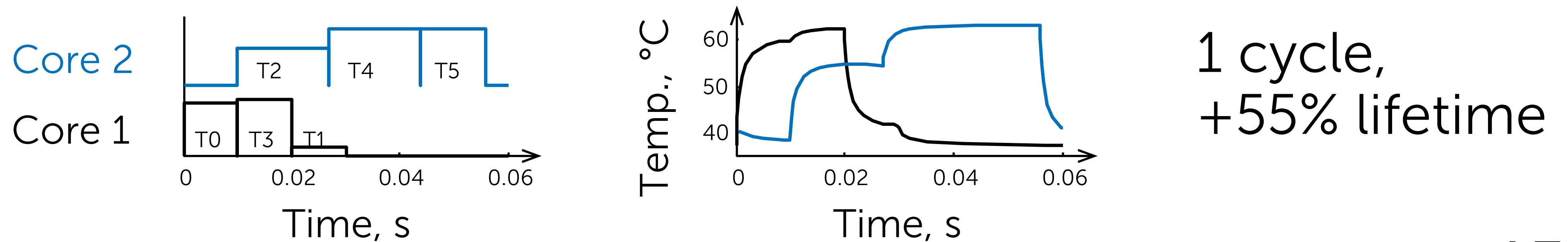
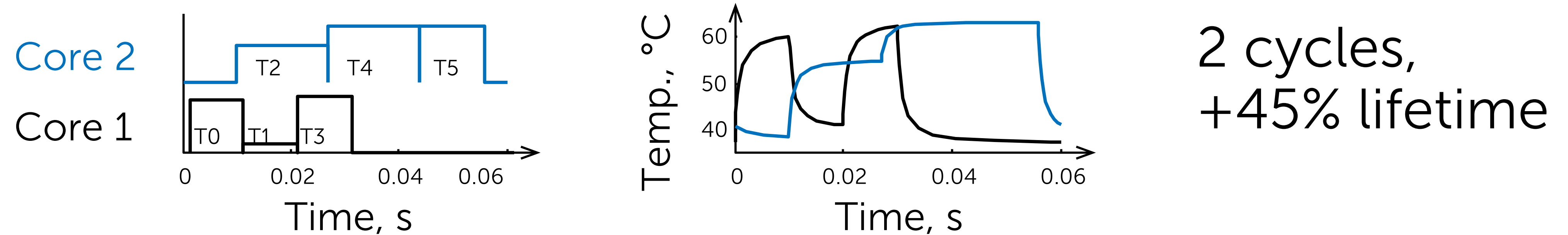
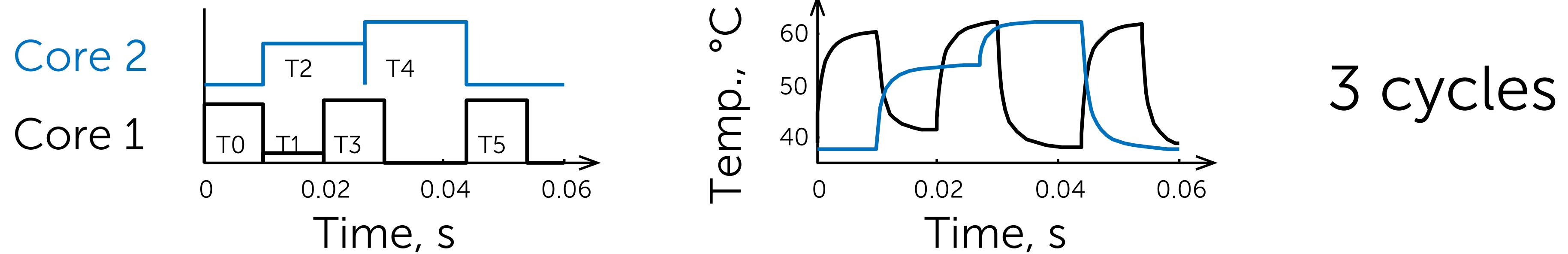


3 cycles



2 cycles,
+45% lifetime

Reliability Optimization



Objective

$$\min_{\mathcal{S}} \mathbb{E}[\text{Energy}]$$

Execution time(\mathcal{S}) < Deadline

$\mathbb{P}(\text{Temperature}(\mathcal{S}) > \text{Maximal}) < p_{\text{burn}}$

$\mathbb{P}(\text{Lifetime}(\mathcal{S}) < \text{Minimal}) < p_{\text{wear}}$

Experimental Setup

- * 2 process parameters
- * 2, 4, 8, 16, and 32 cores
- * 40, 80, 160, 320, and 640 tasks
- * 10 test cases per pair cores/tasks

Probabilistic vs. Deterministic

Cores	Prob., min	Det., min	Fail, %
2	1	1	40
4	5	2	60
8	17	4	70
16	56	8	100
32	300	9	100

Thank you! Questions?

<https://users.ece.cmu.edu/~iukhov>