Adaptive Interpolation for Electronic-System Design

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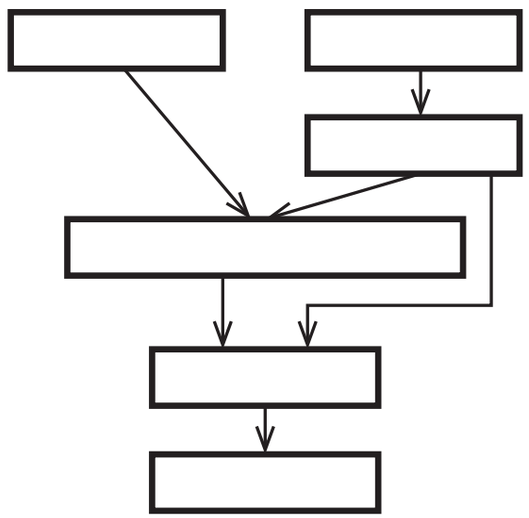
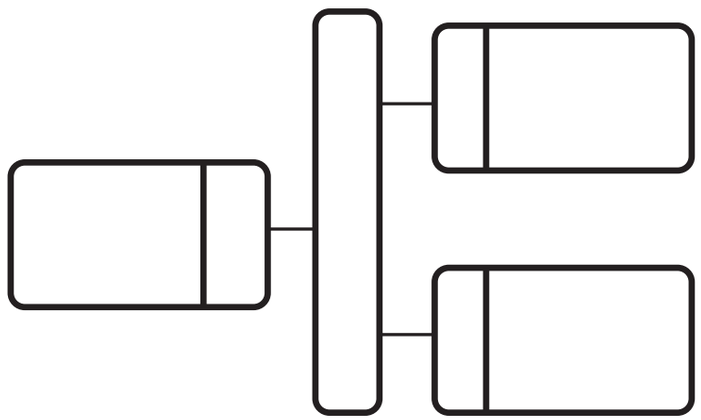
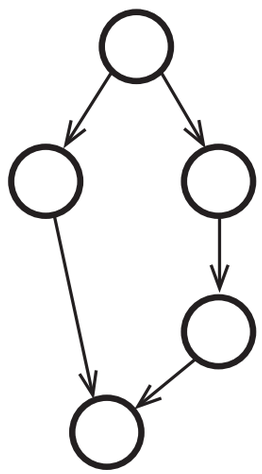
Outline

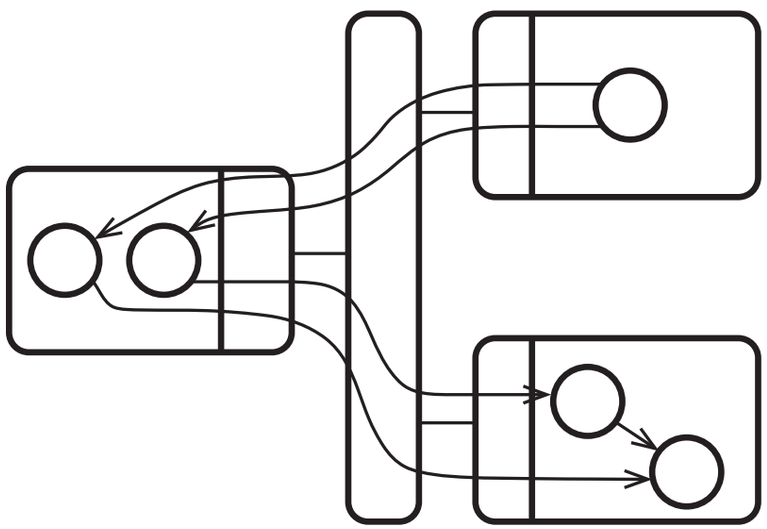
- Motivation
- Interpolation

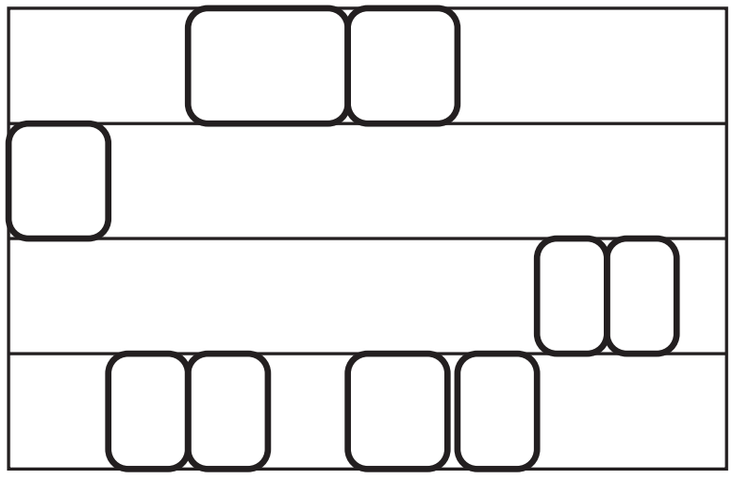
Electronic-System Design

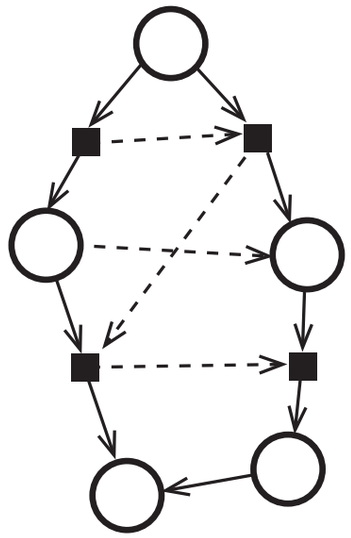




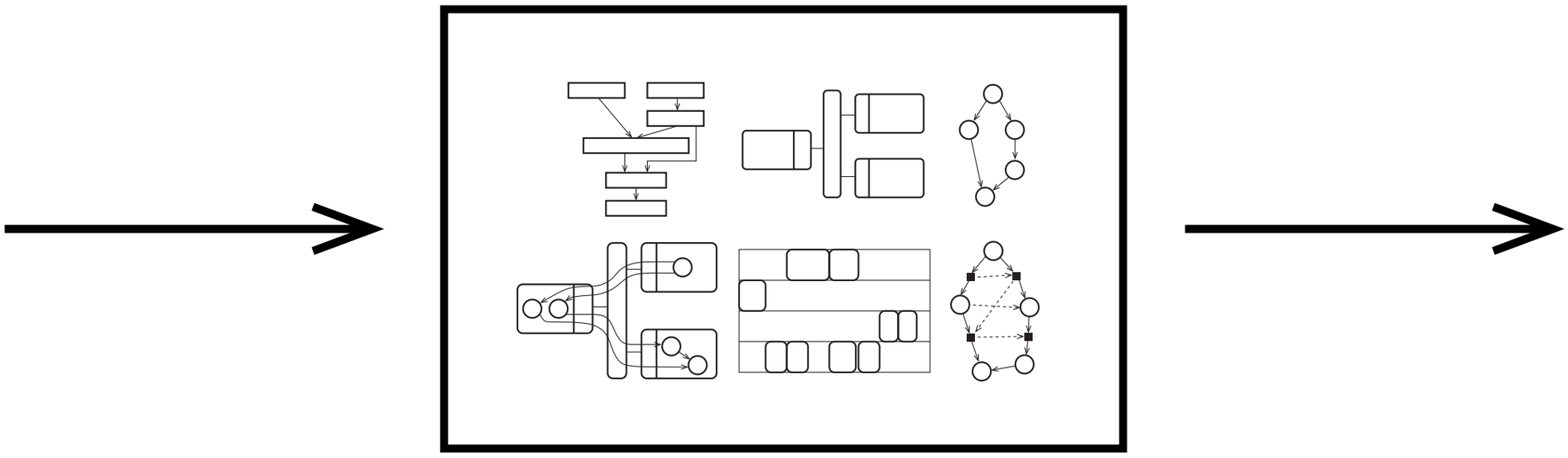








Electronic-System Design



Electronic-System Design

$$f(\mathbf{x})$$

Questions

- Range of outputs?
- Most influential inputs?
- Input values to agree with data?
- Probability distribution of outputs?

Optimization

$$\min_{\mathbf{x}} f(\mathbf{x})$$

Integration

$$\int f(\mathbf{x})w(\mathbf{x})d\mathbf{x}$$

Inference

$$p(\mathbf{x}|\mathcal{D}) \propto p(\mathcal{D}|\mathbf{x})p(\mathbf{x})$$

Evaluation

$$f(\mathbf{x})$$

Curse of Dimensionality

1	10	10 seconds
2	100	2 minutes
3	1,000	16 minutes
4	10,000	3 hours
5	100,000	1 day
6	1,000,000	2 weeks
...	...	...
20	100,000,000,000, 000,000,000	3 trillion years (200 × the universe)

Curse of Complexity

blackscholes	1	1 hour
bodytrack	1	2 hours
canneal	1	2 hours
dedup	1	4 hours
facesim	1	13 hours
ferret	1	3 hours
...	...	...
x264	1	3 hours

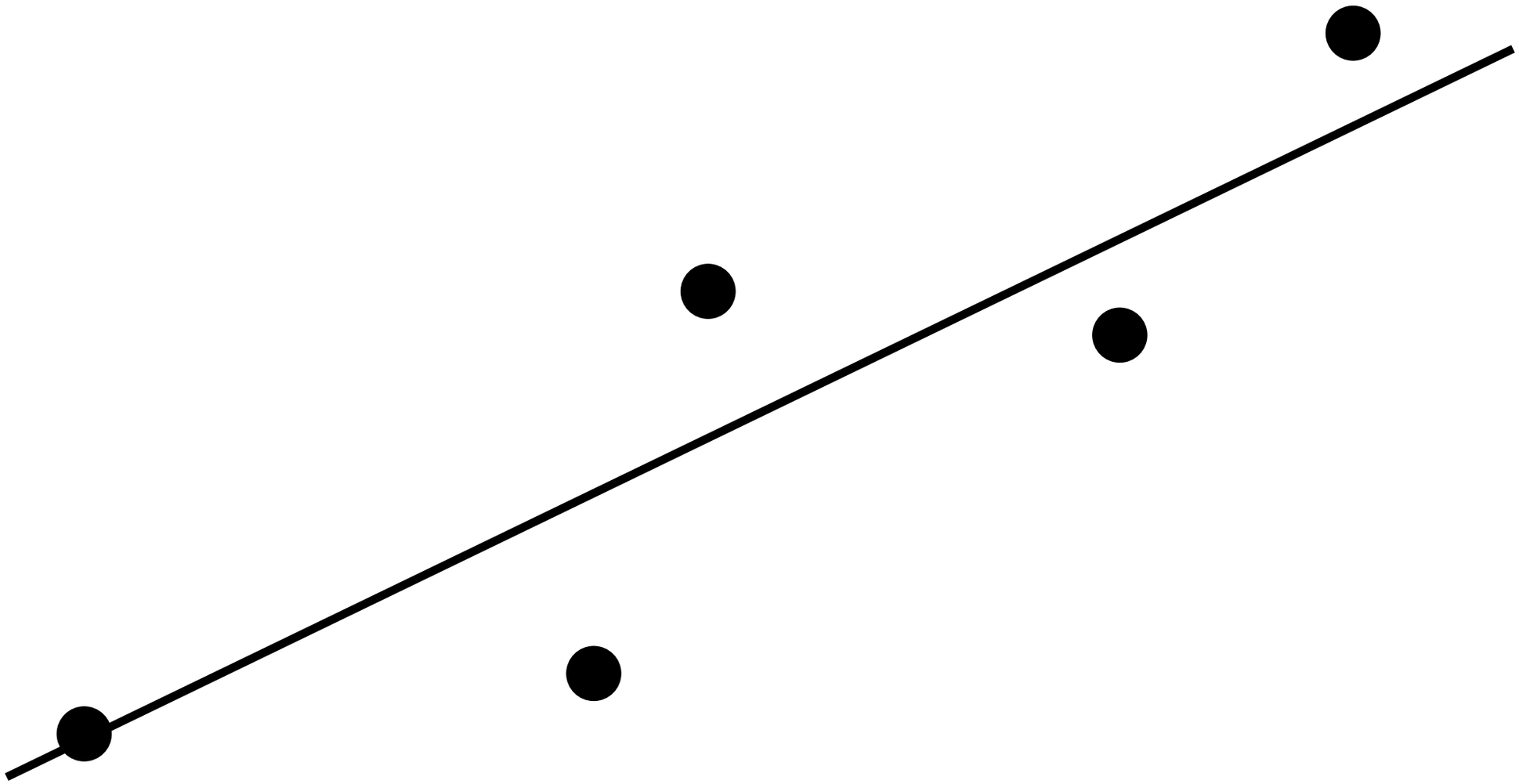
Approximation

$$f(\mathbf{x}) \approx f(\mathbf{x})$$

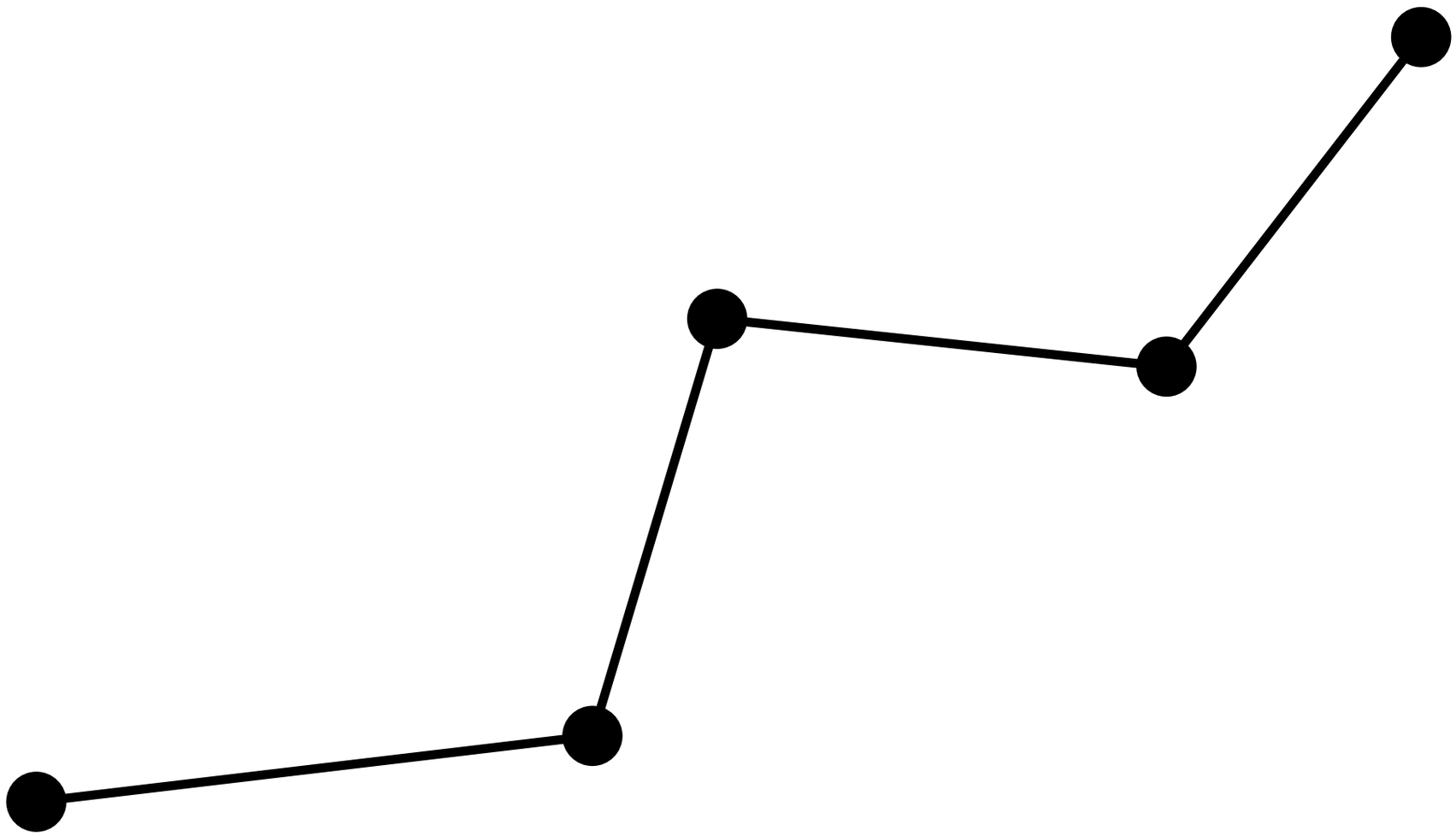
Approximation

- Regression
- Interpolation

Regression



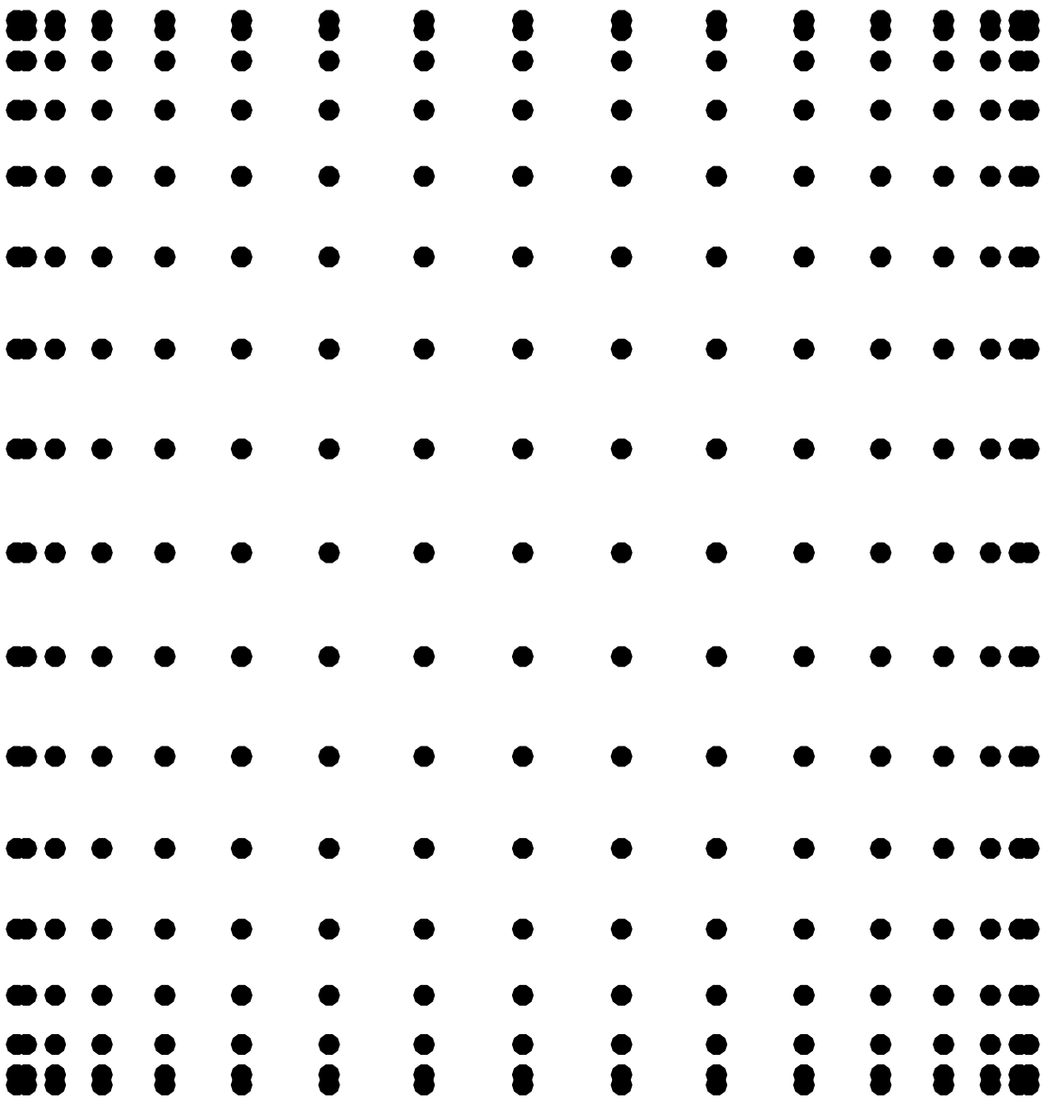
Interpolation

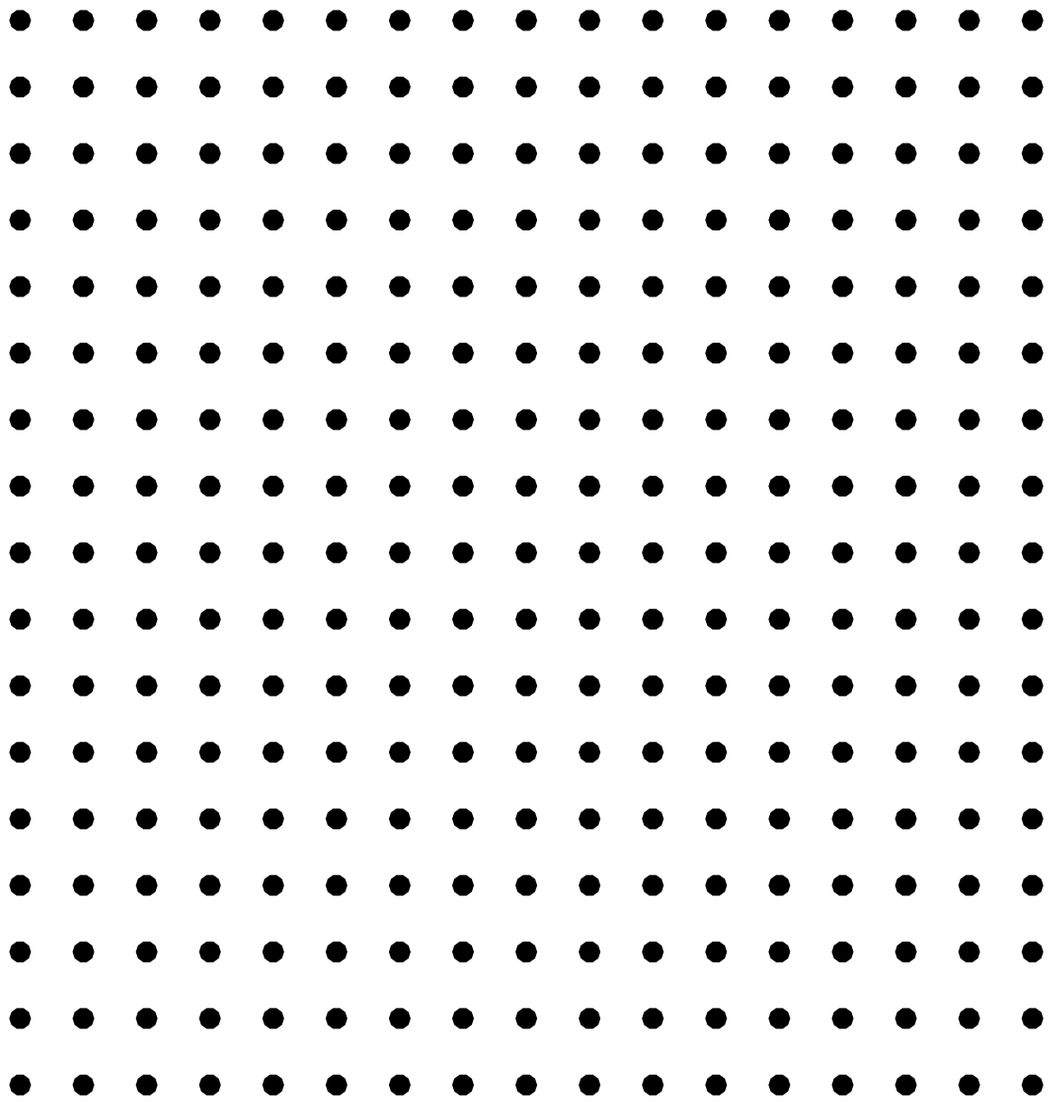


Interpolation

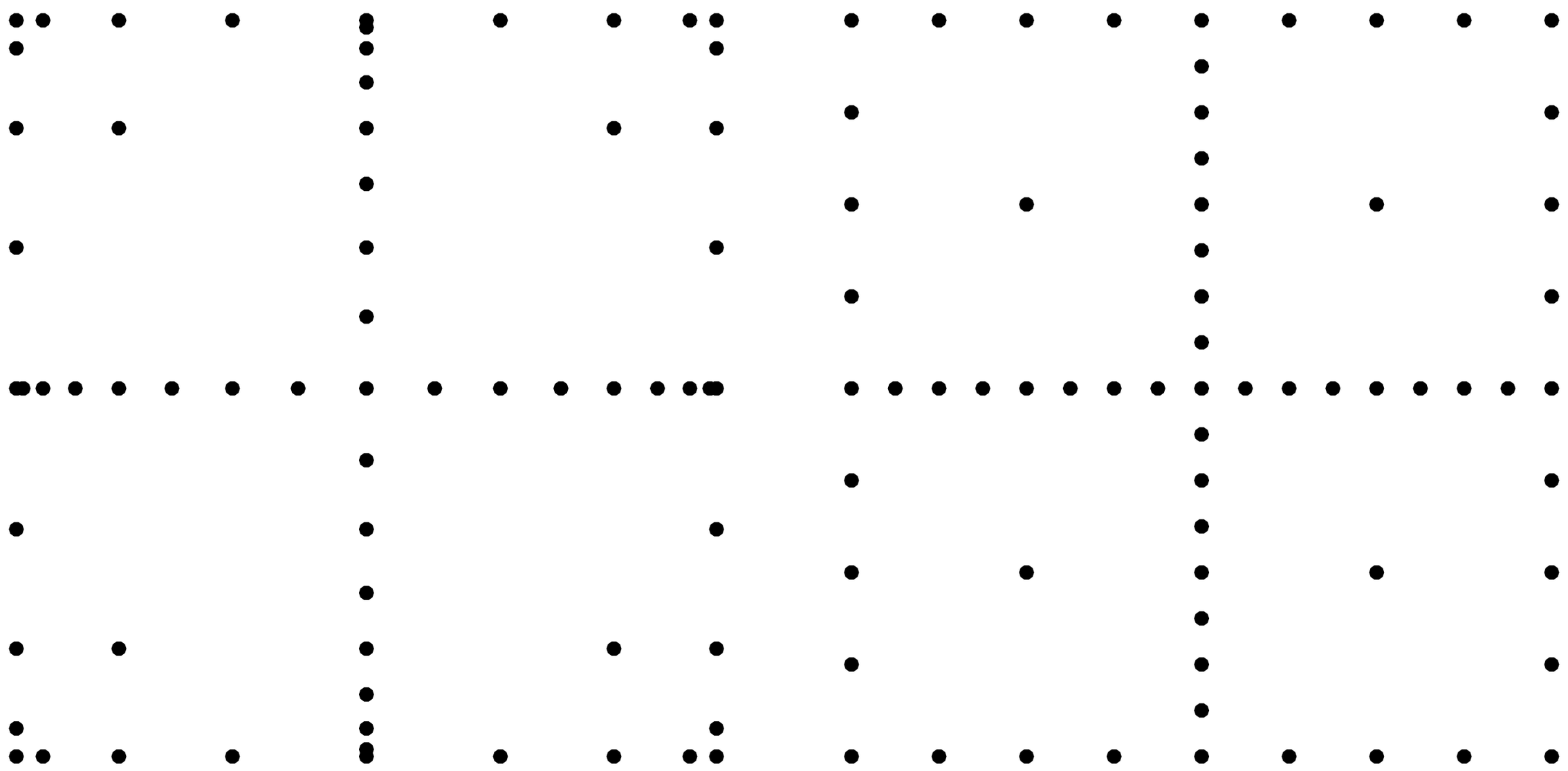
- Grid
- Basis
- Structure
- Adaptivity

Full-Tensor Grid

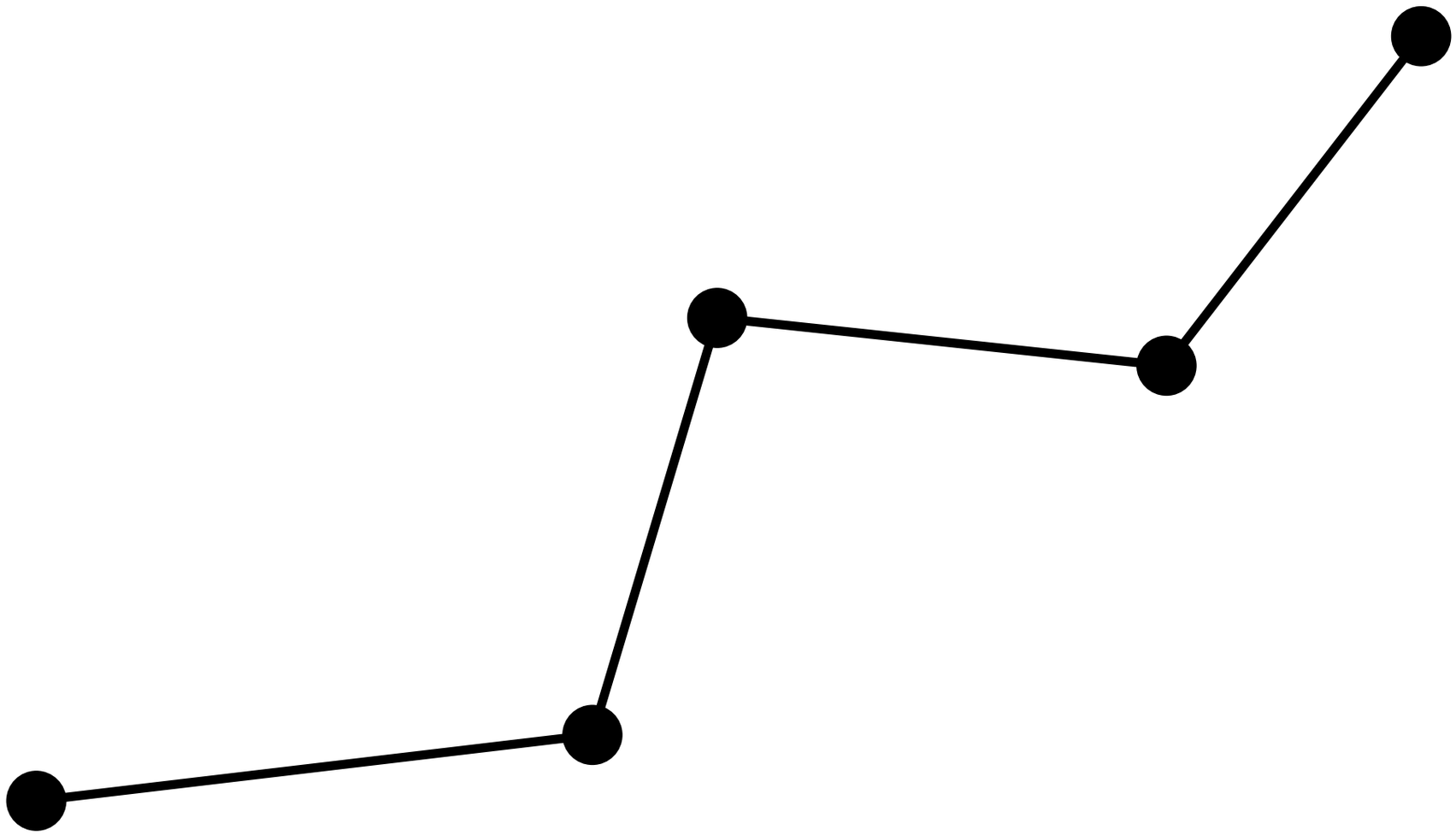




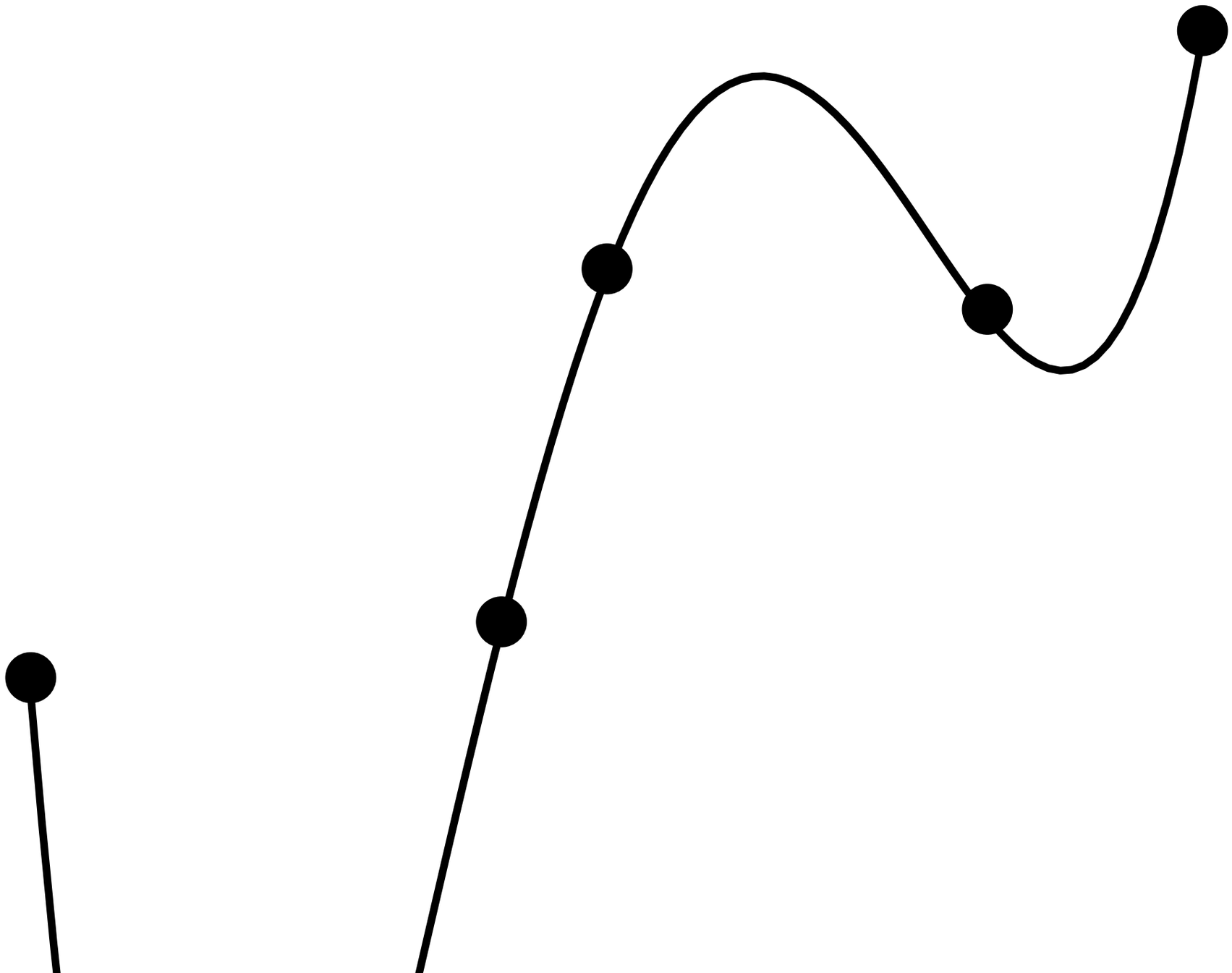
Smolyak (Sparse) Grid



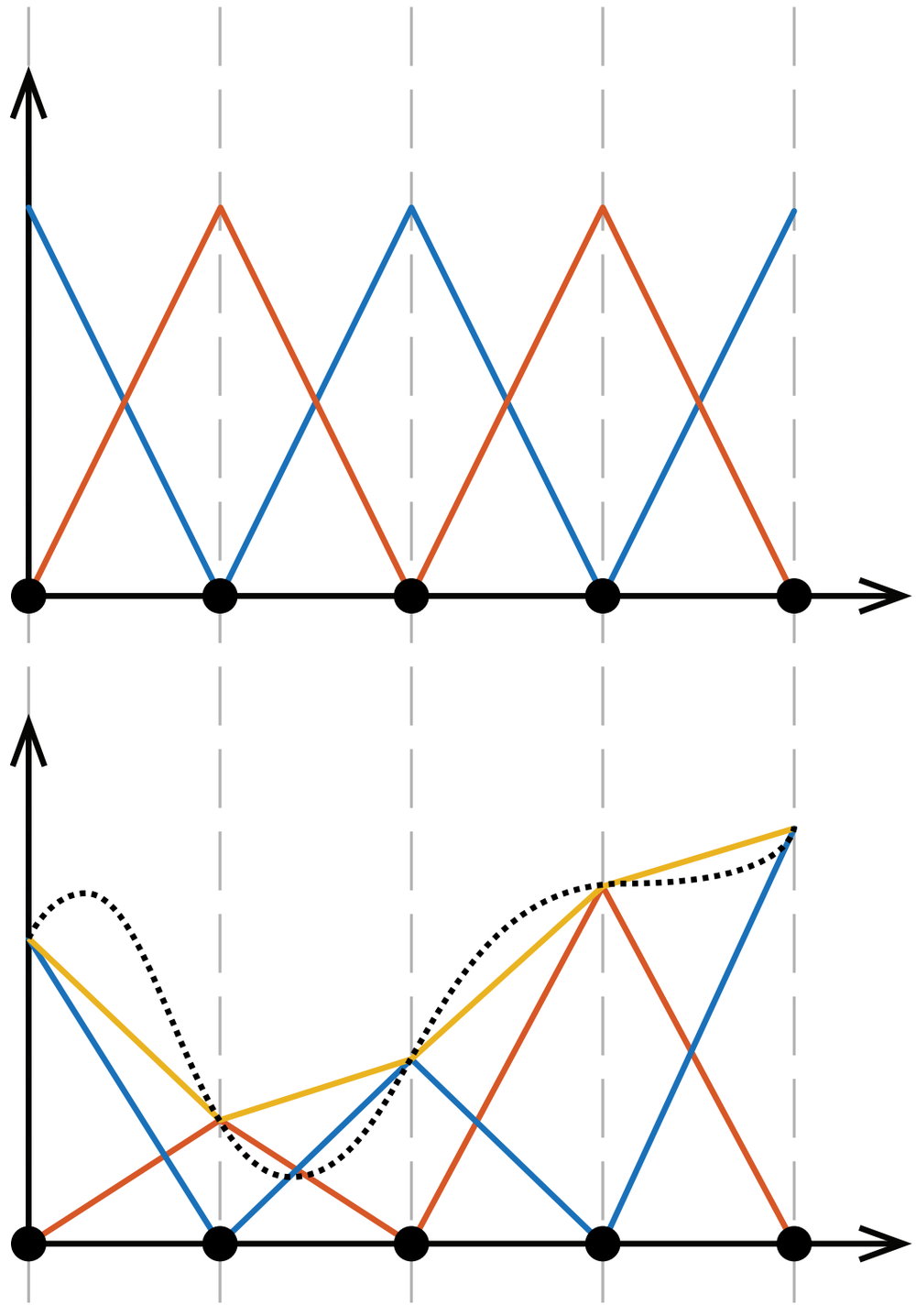
Piecewise Linear Basis



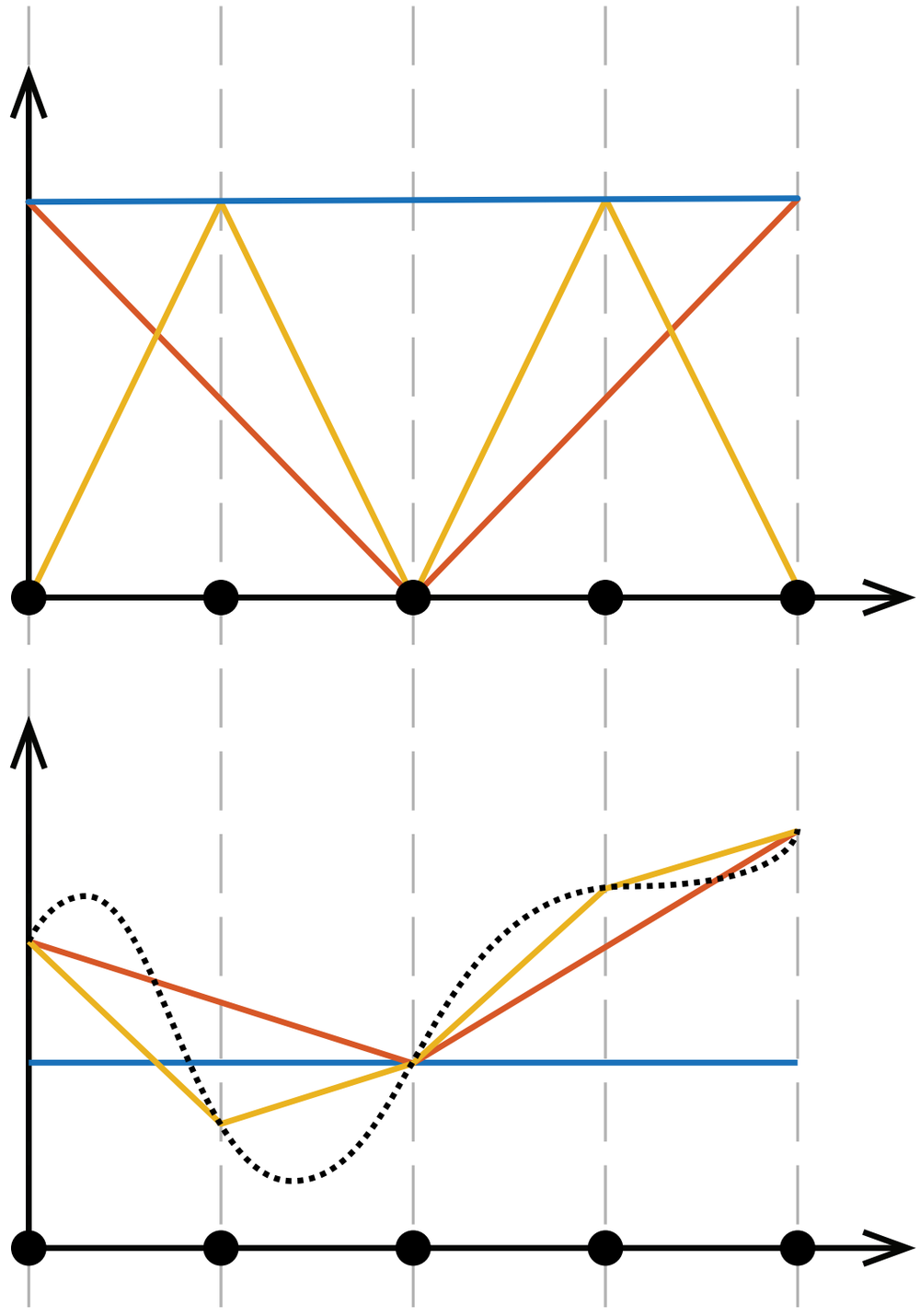
Lagrange Basis



Nodal Structure



Hierarchical Structure



Multiindex

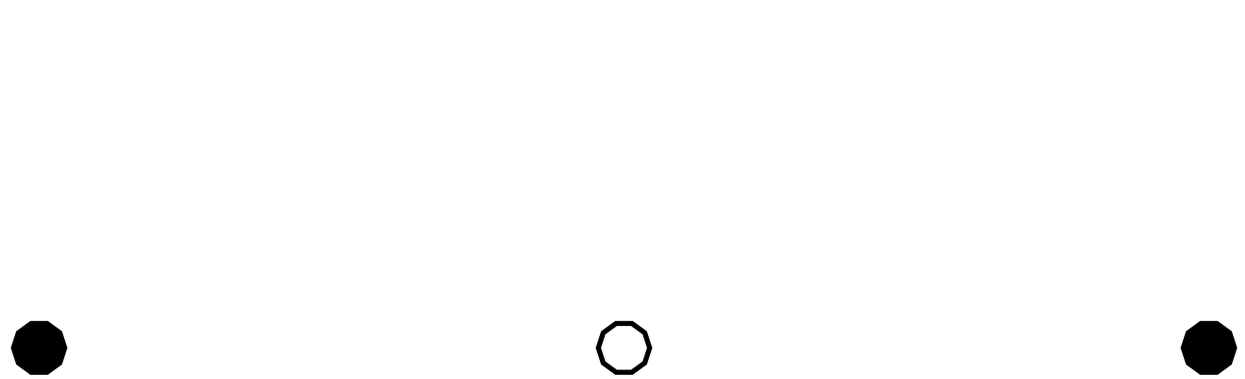
$$\mathbf{i} = (i_1, \ldots, i_n)$$

Nestedness

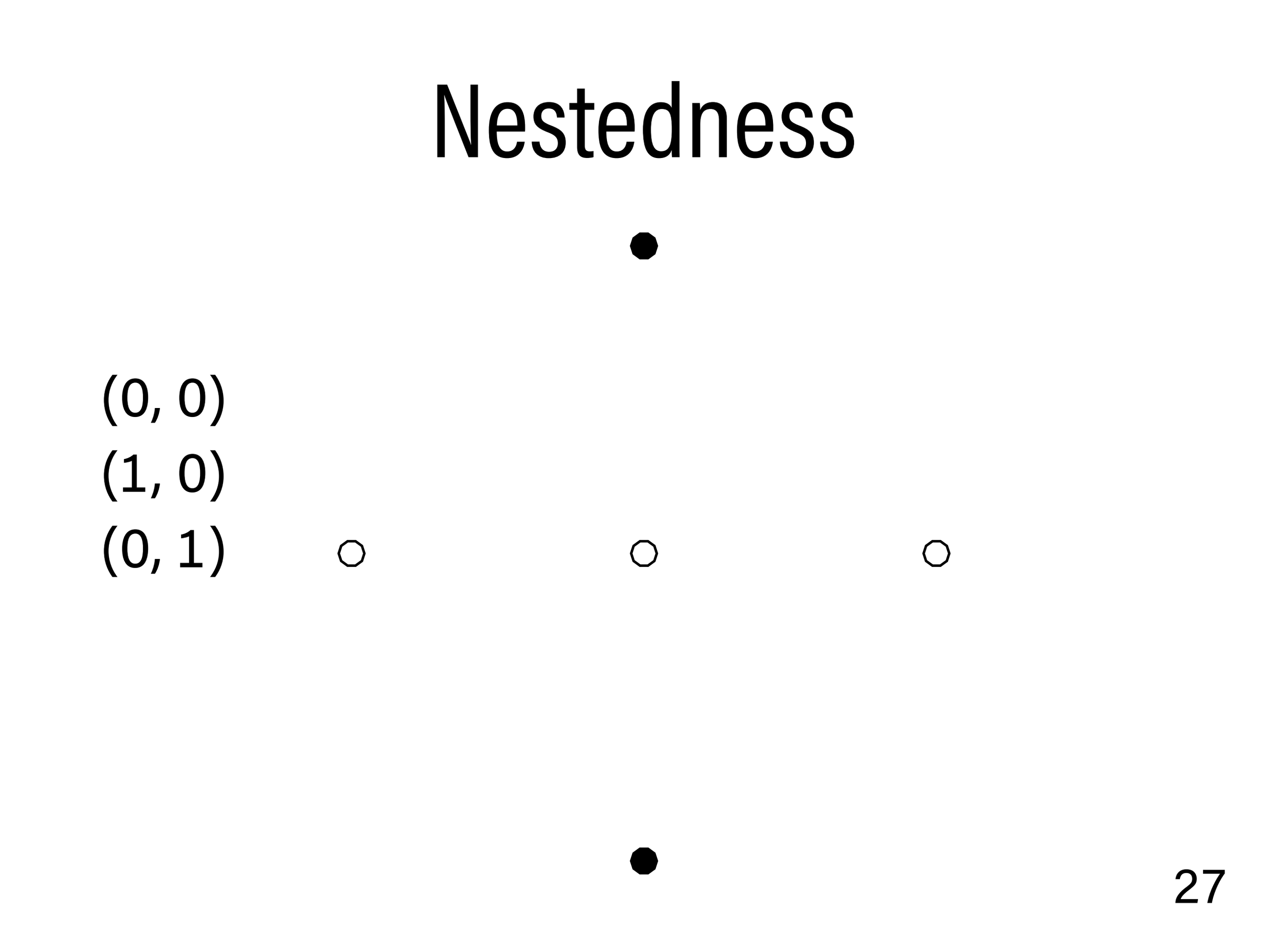
(0, 0)

Nestedness

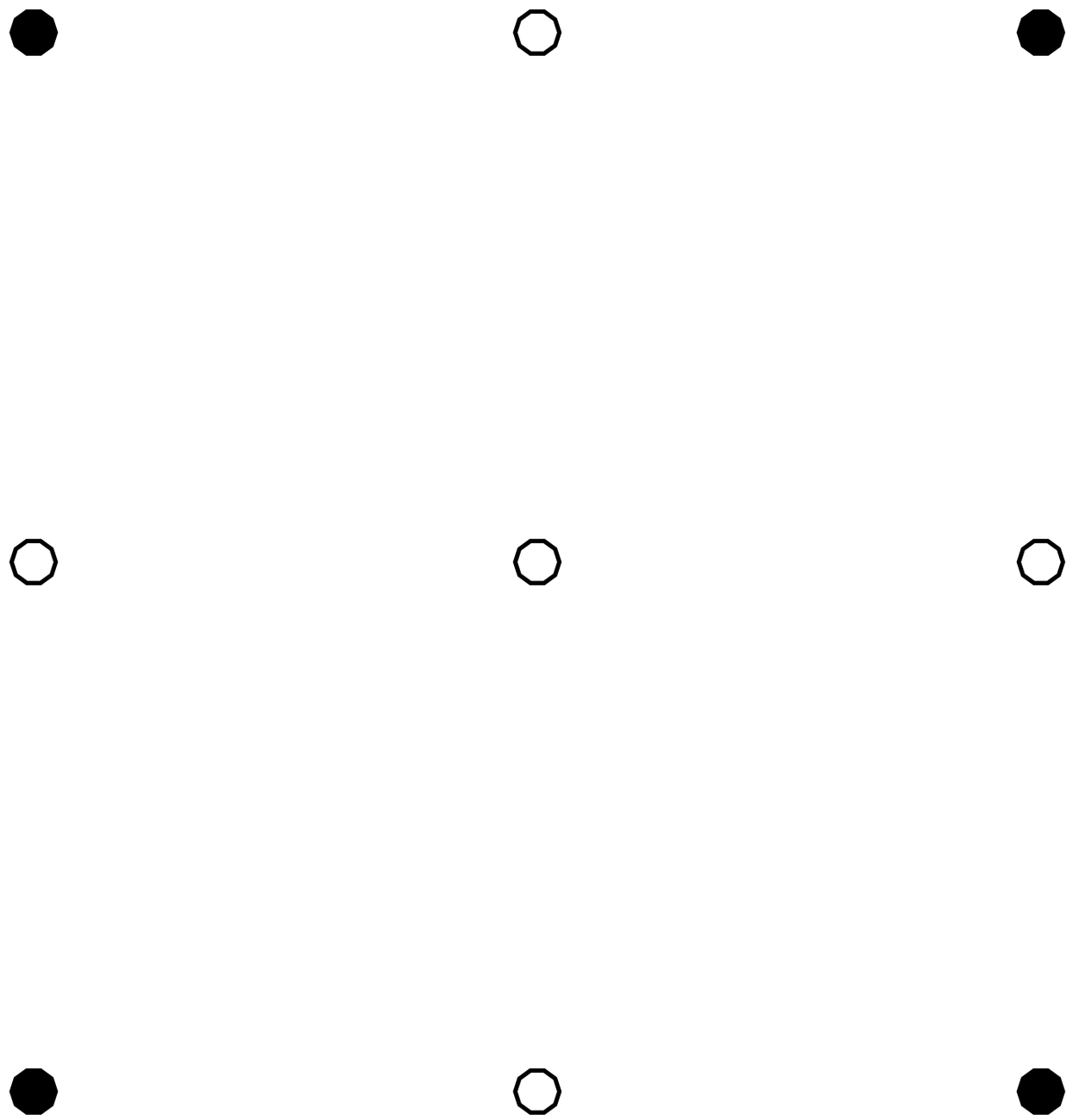
(0, 0)
(1, 0)



Nestedness



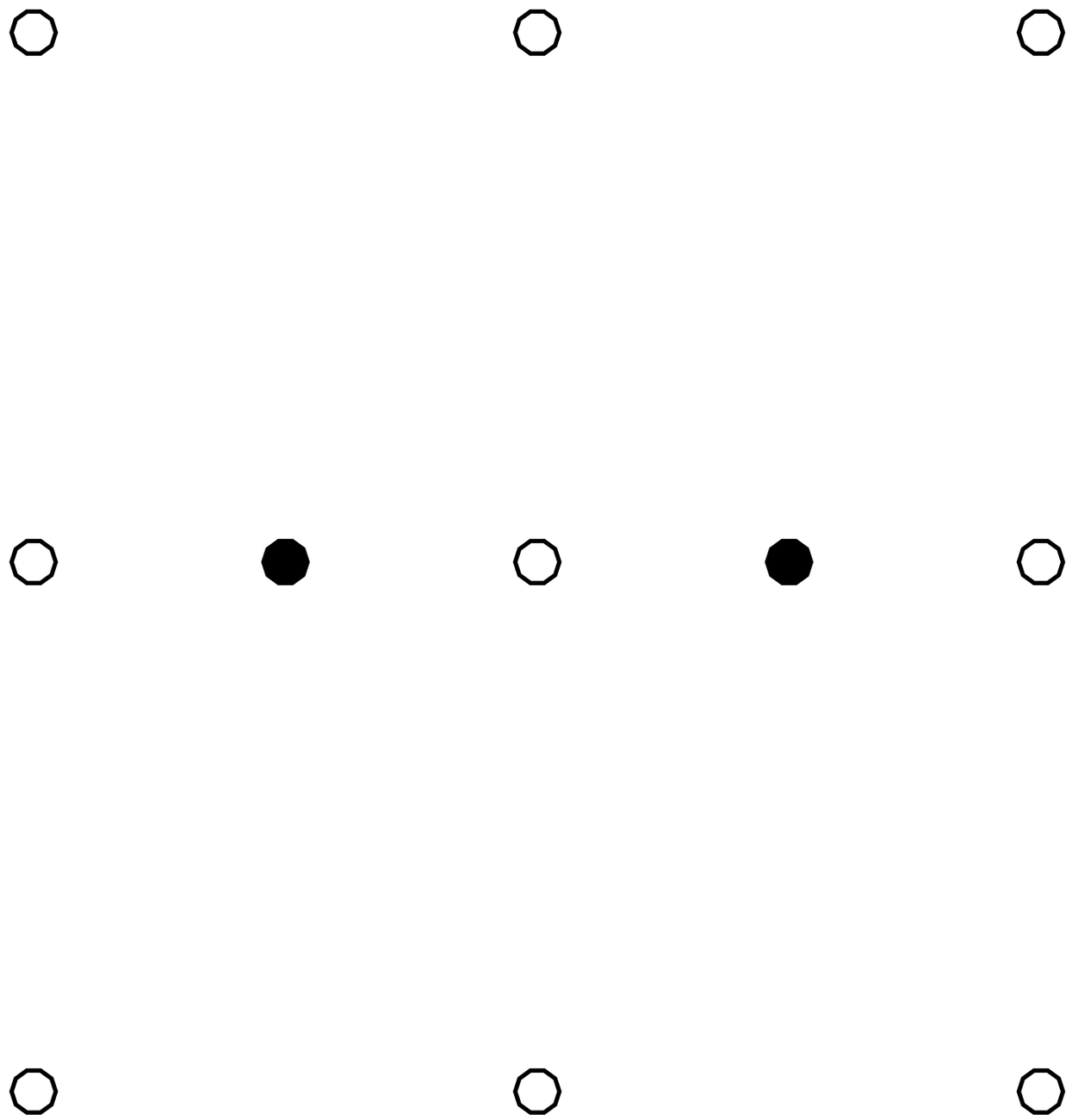
Nestedness



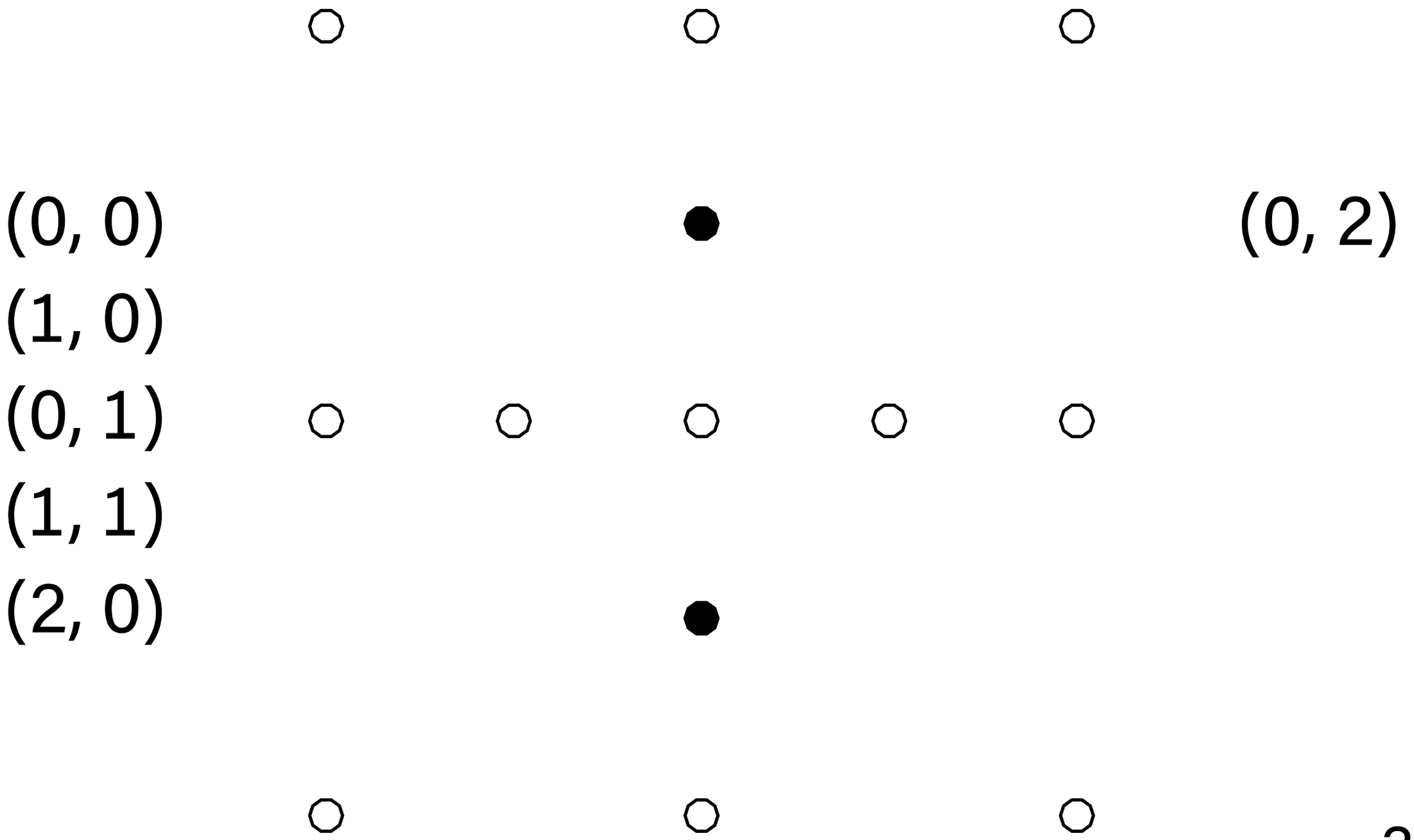
(0, 0)
(1, 0)
(0, 1)
(1, 1)

Nestedness

(0, 0)
(1, 0)
(0, 1)
(1, 1)
(2, 0)



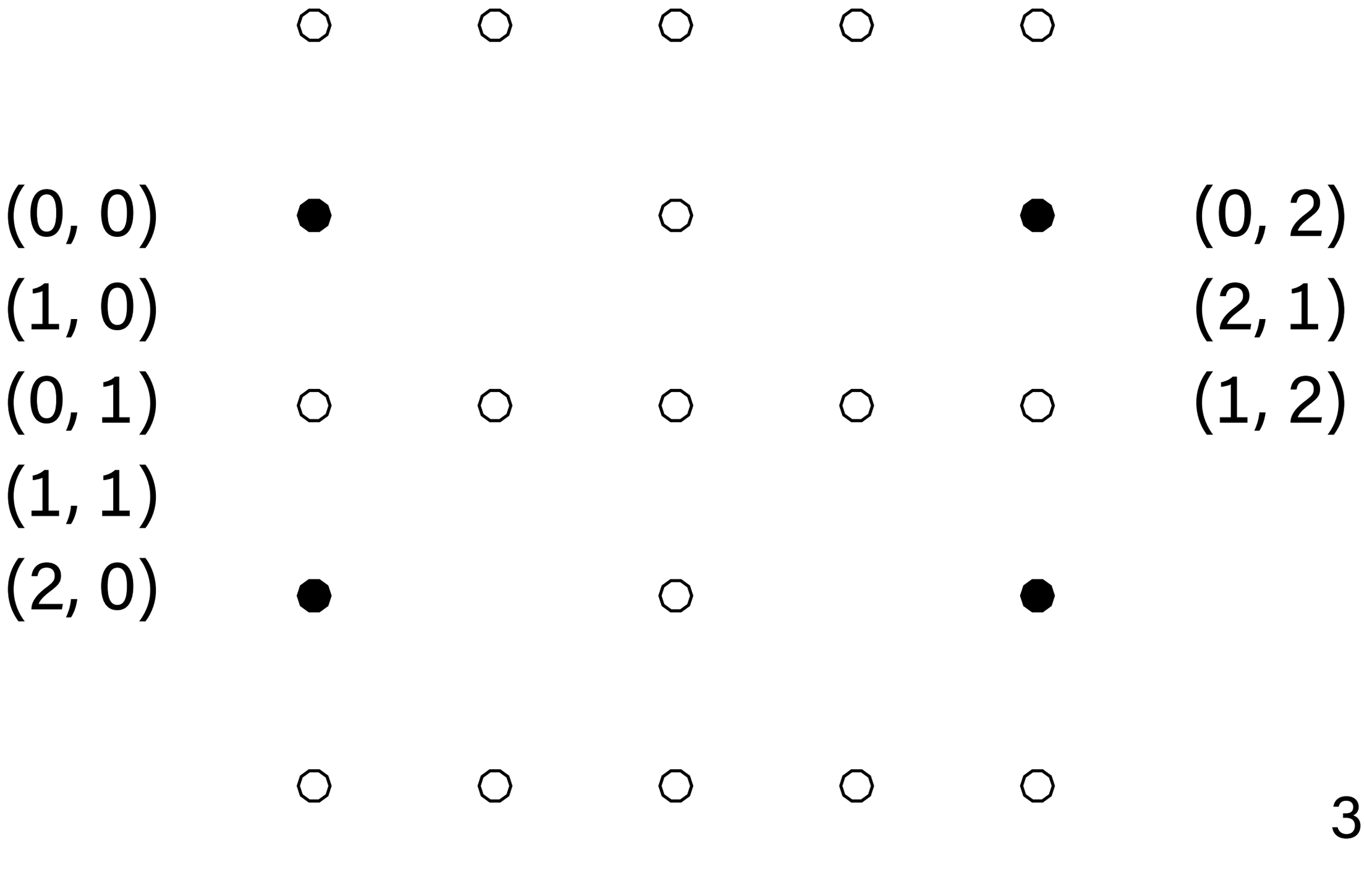
Nestedness



Nestedness

(0, 0)
(1, 0)
(0, 1)
(1, 1)
(2, 0)
(0, 2)
(2, 1)

Nestedness



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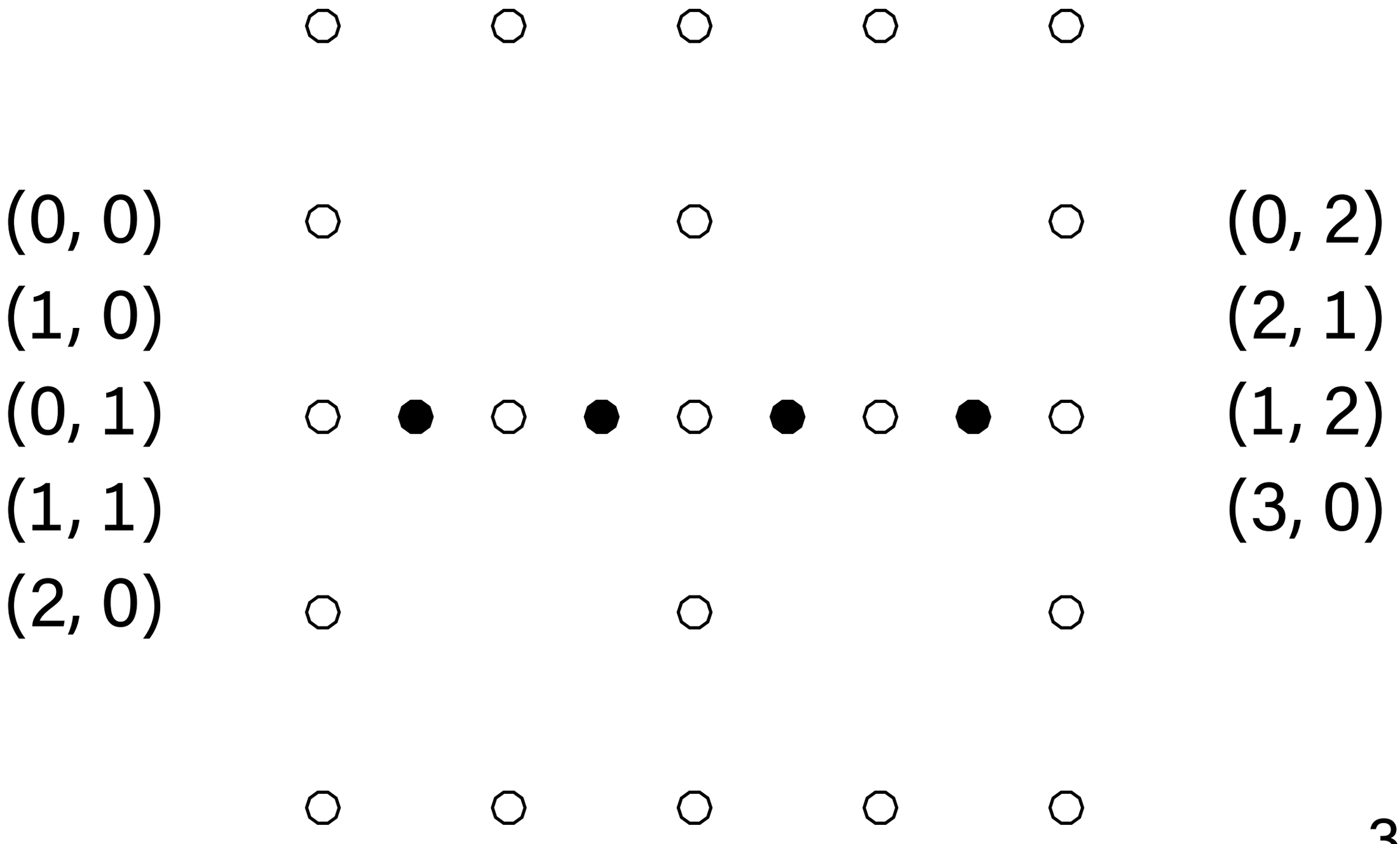
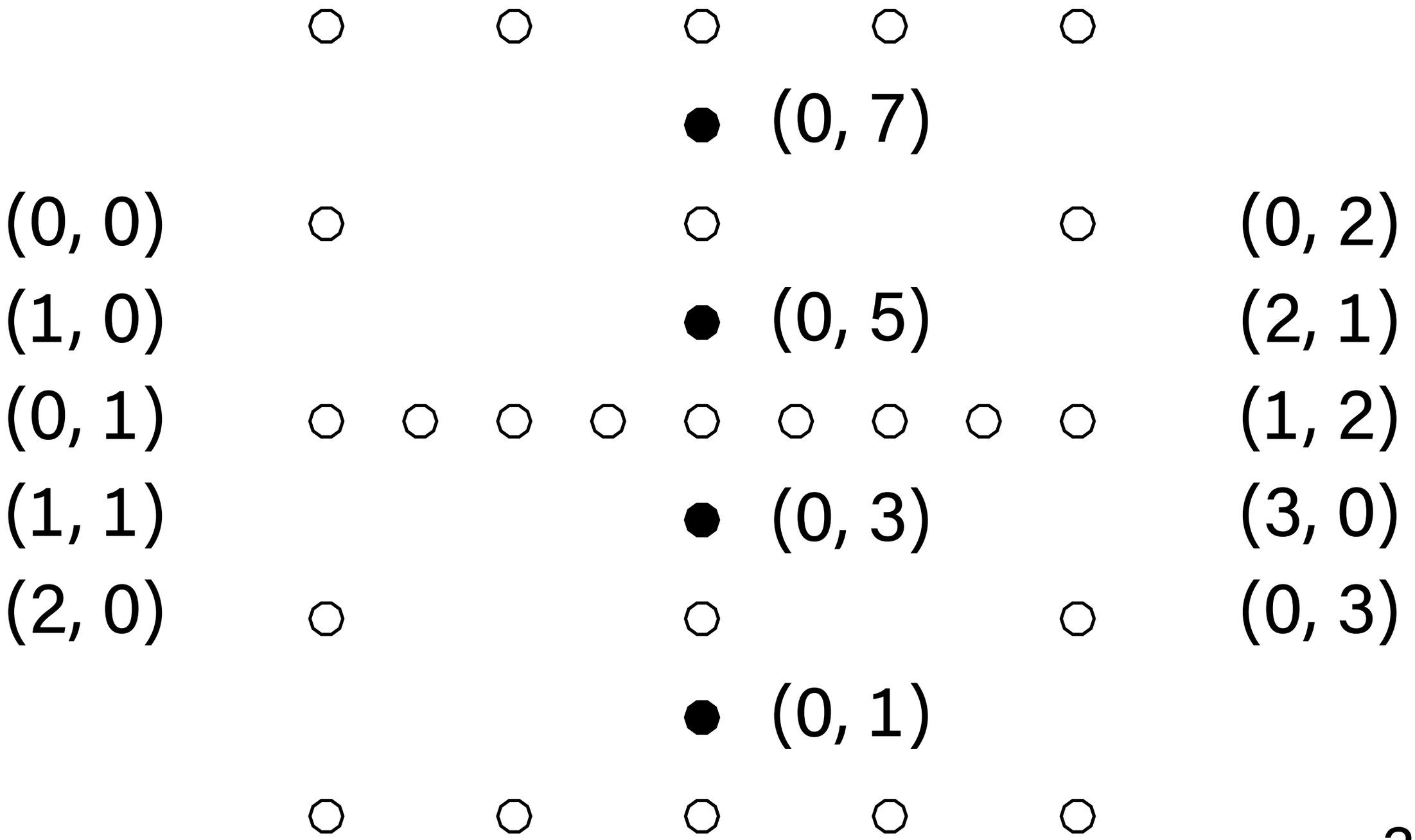
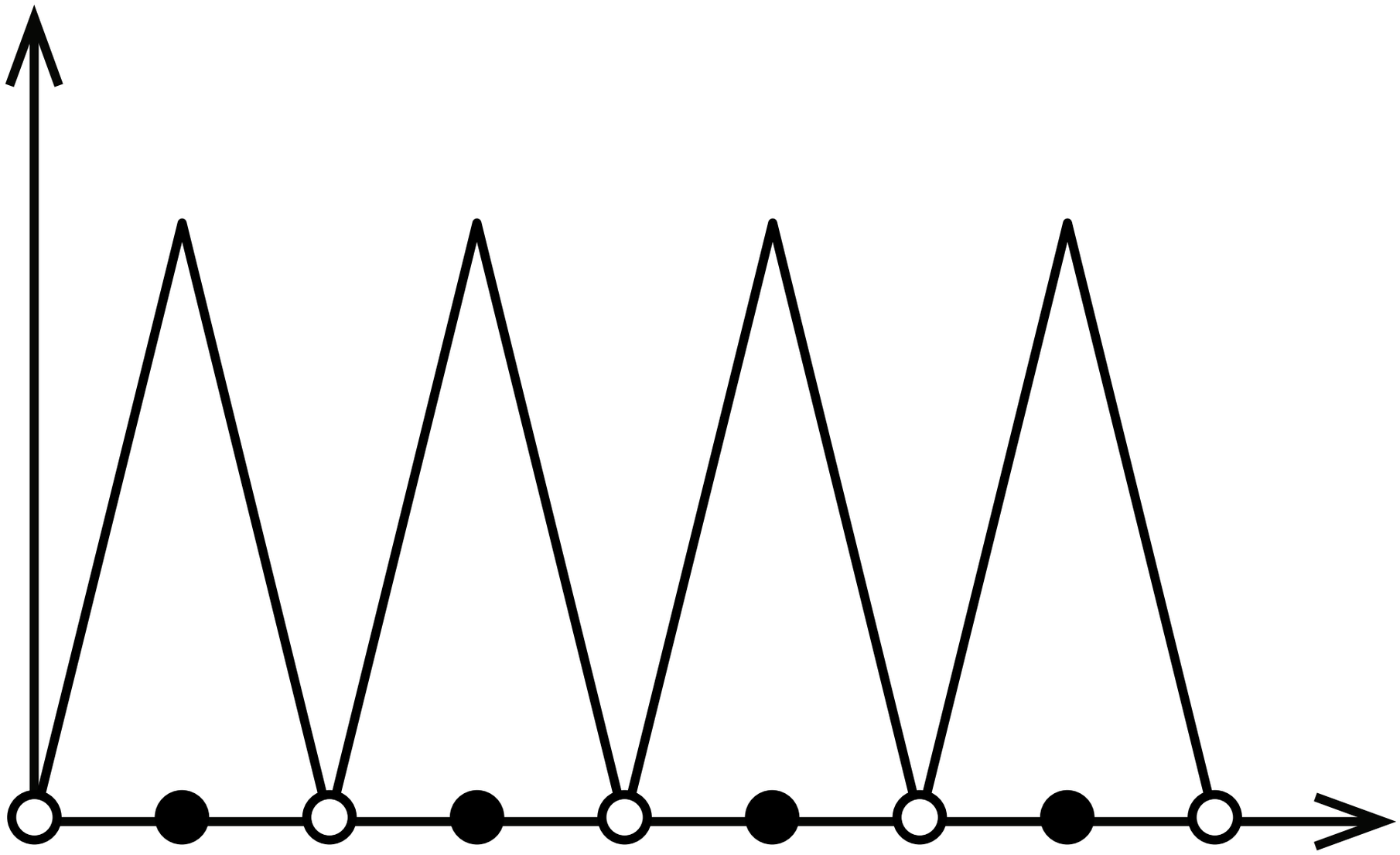


Nestedness



Locality



Surrogate

$$\tilde{f}(\mathbf{x}) = \sum_{\mathbf{i}\in\mathcal{I}} \sum_{\mathbf{j}\in\mathcal{J}_\mathbf{i}} \Delta f(\mathbf{x_{ij}}) e_{\mathbf{ij}}(\mathbf{x})$$

Surrogate

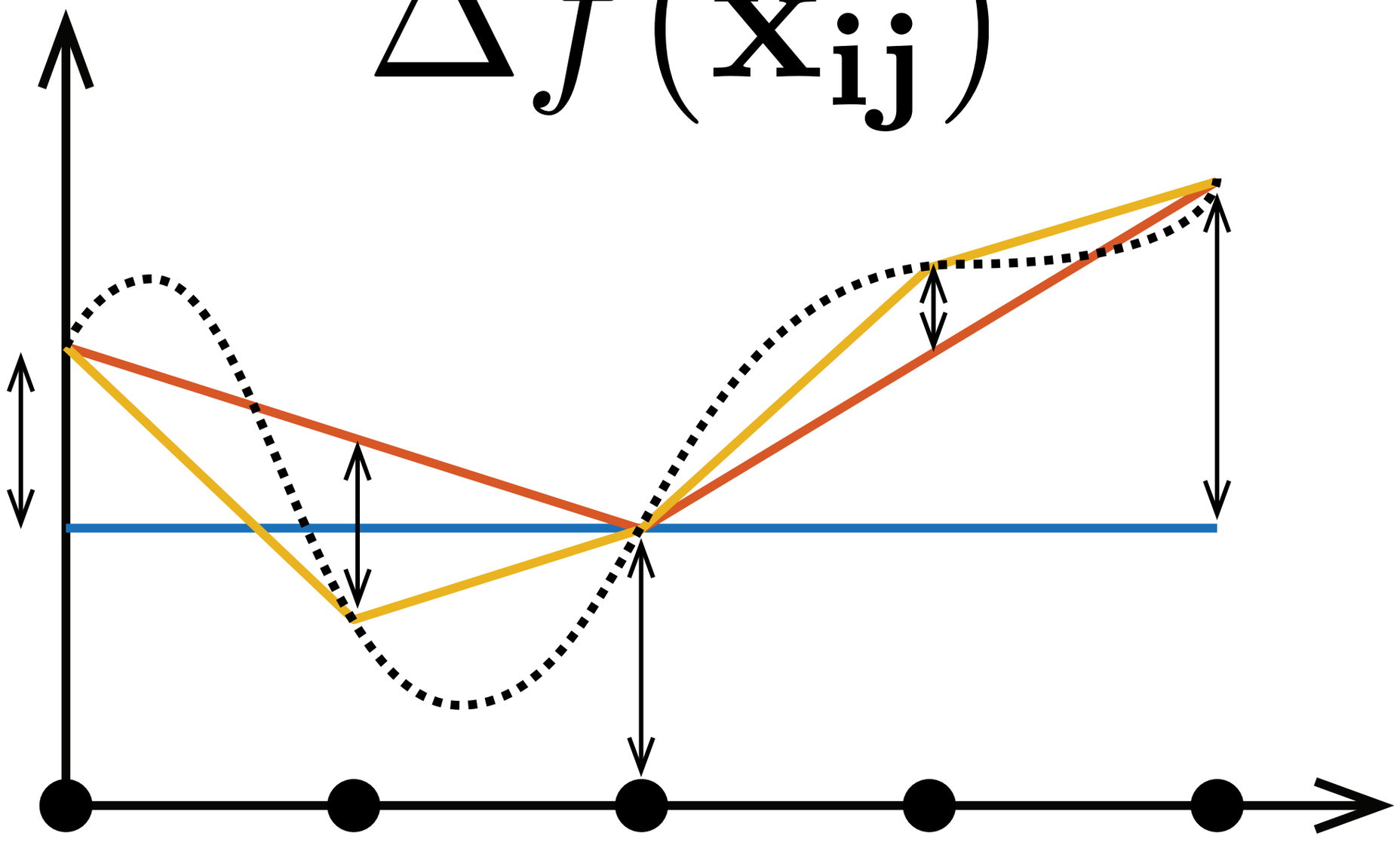
$$\tilde{f}(\mathbf{x}) = \sum_{\mathbf{k} \in \mathcal{K}} \Delta f(\mathbf{x_k}) e_{\mathbf{k}}(\mathbf{x})$$

Adaptivity

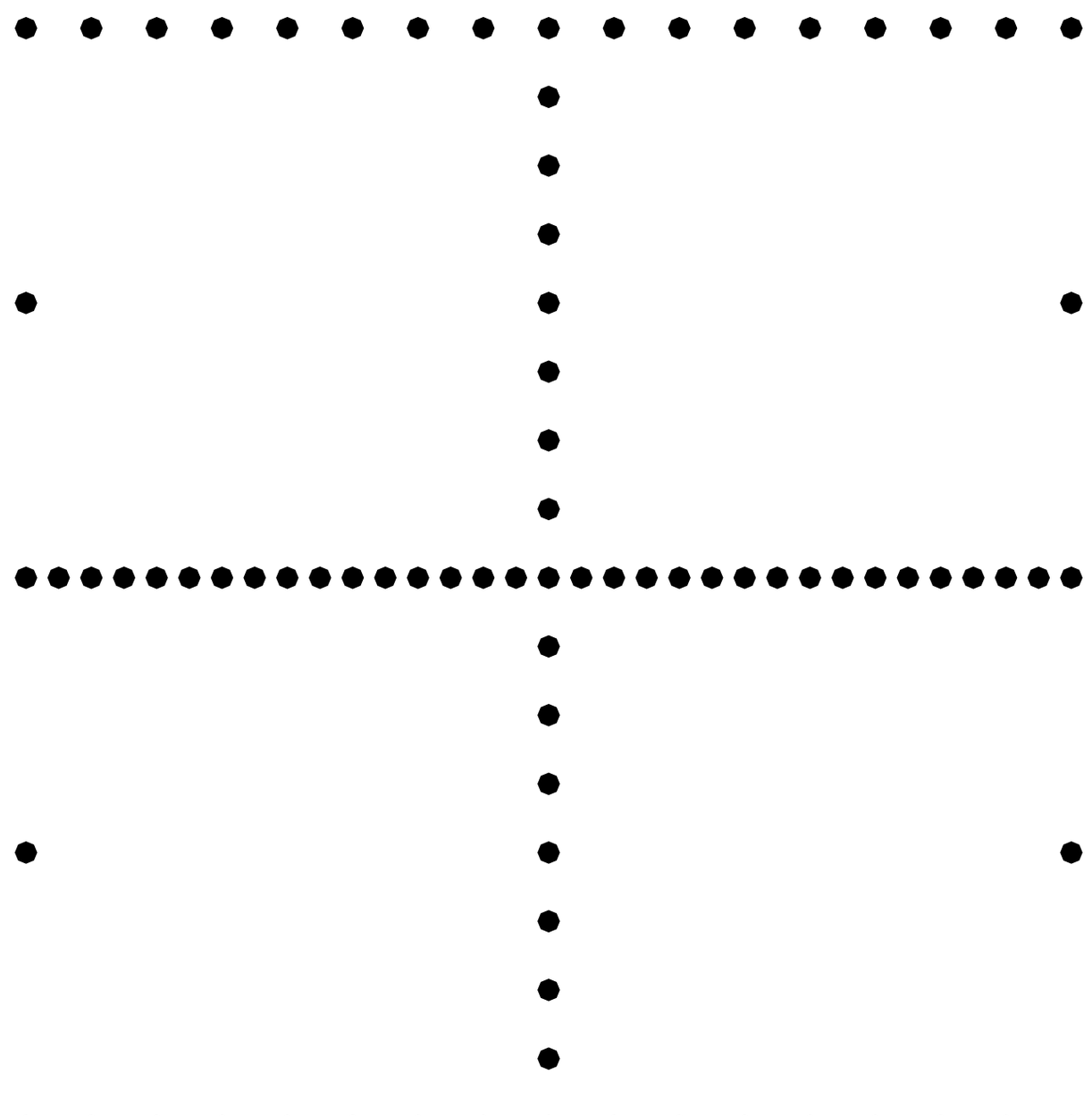
- Global
- Local
- Hybrid

Surplus

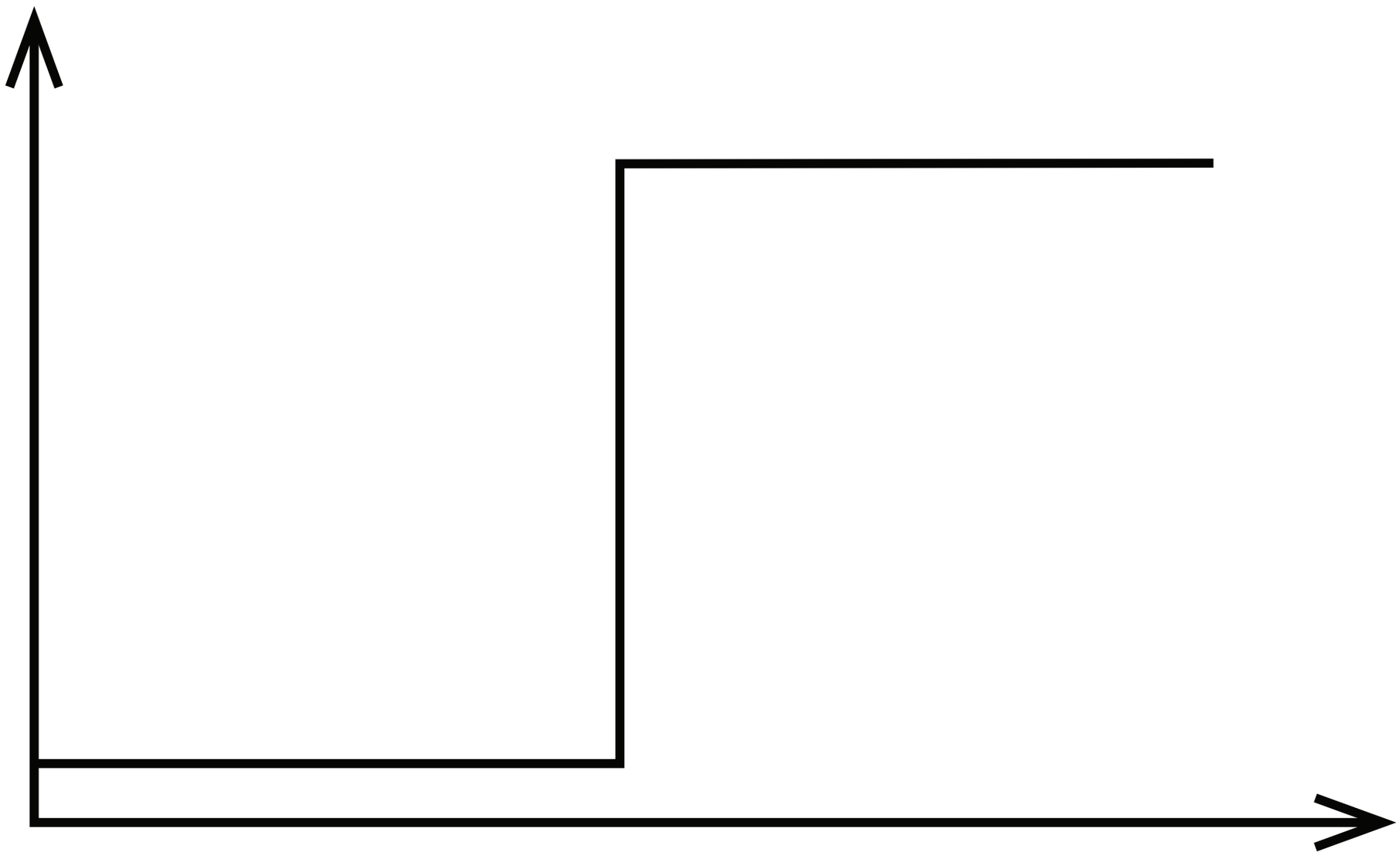
$$\Delta f(\mathbf{x_{ij}})$$



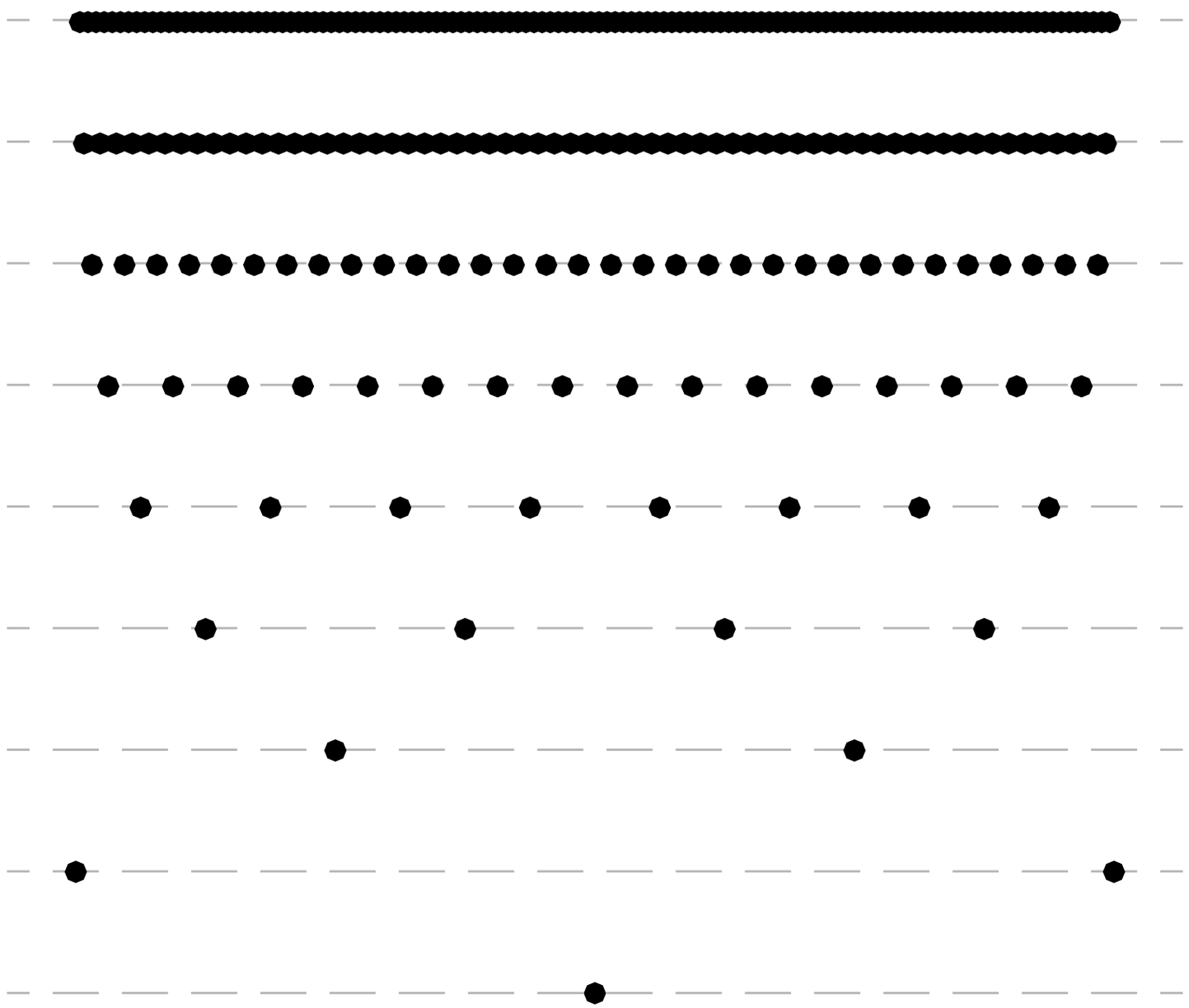
Global Adaptivity



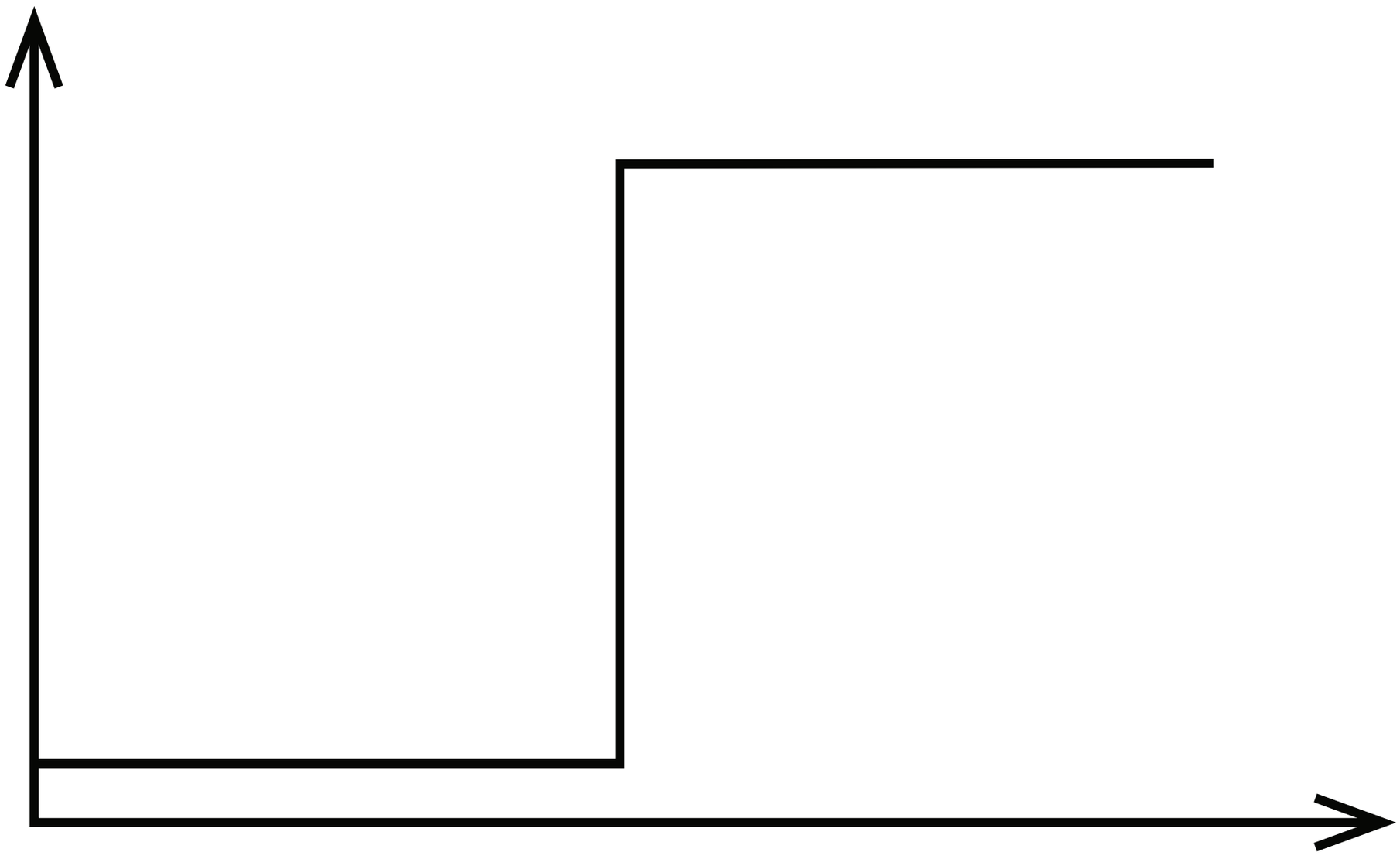
Step Function



Step Function

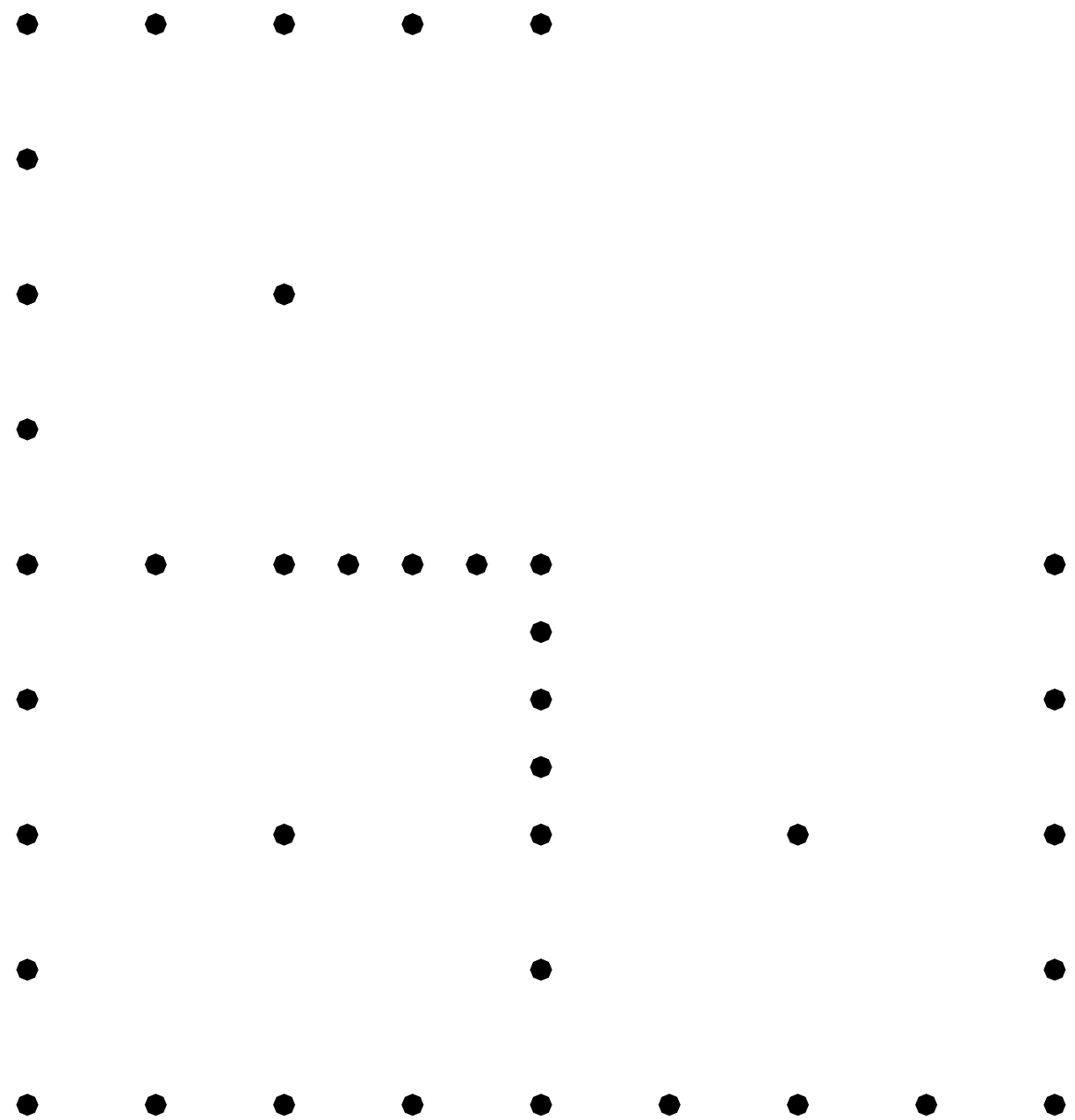


Where Is the Action?

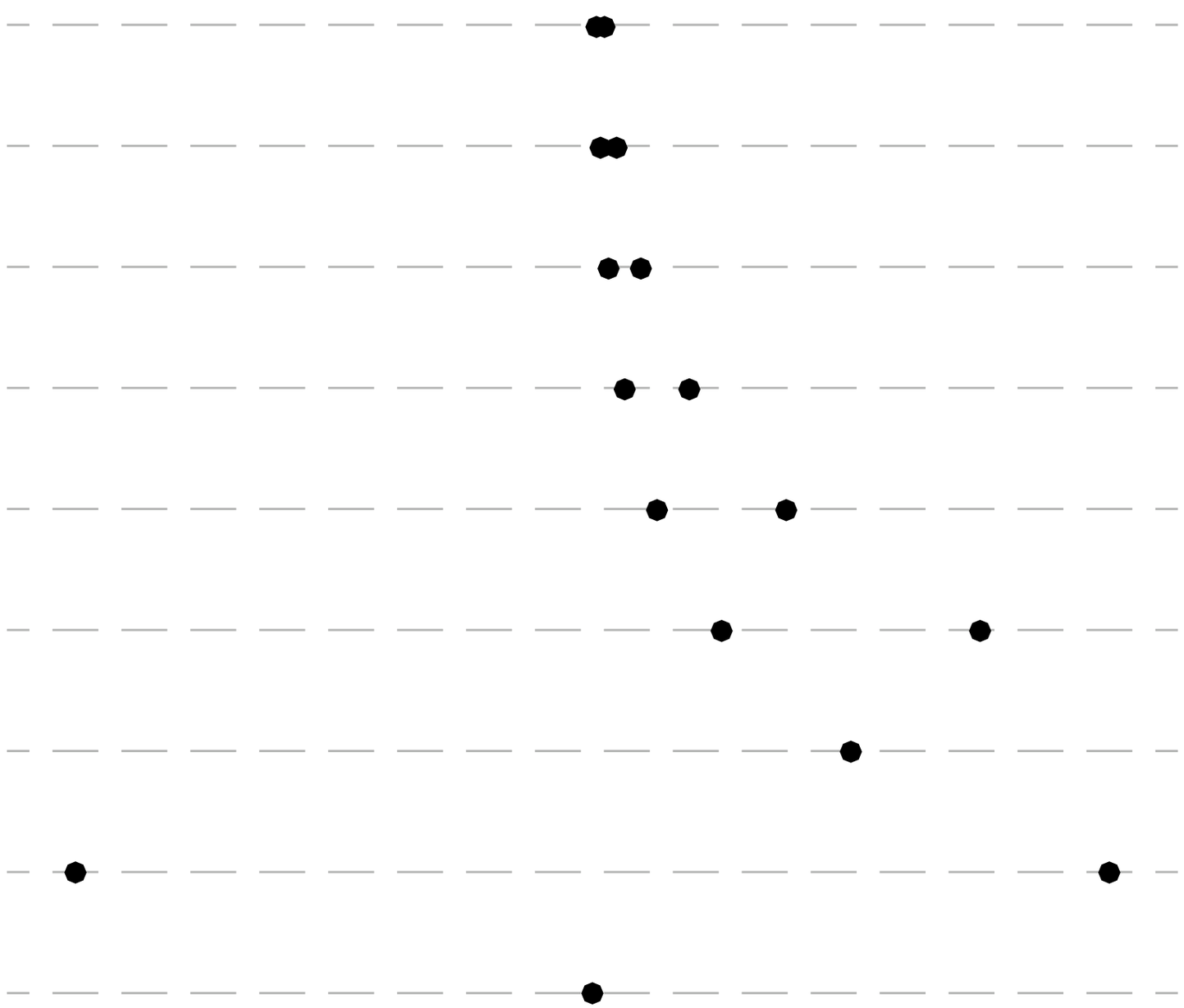


?

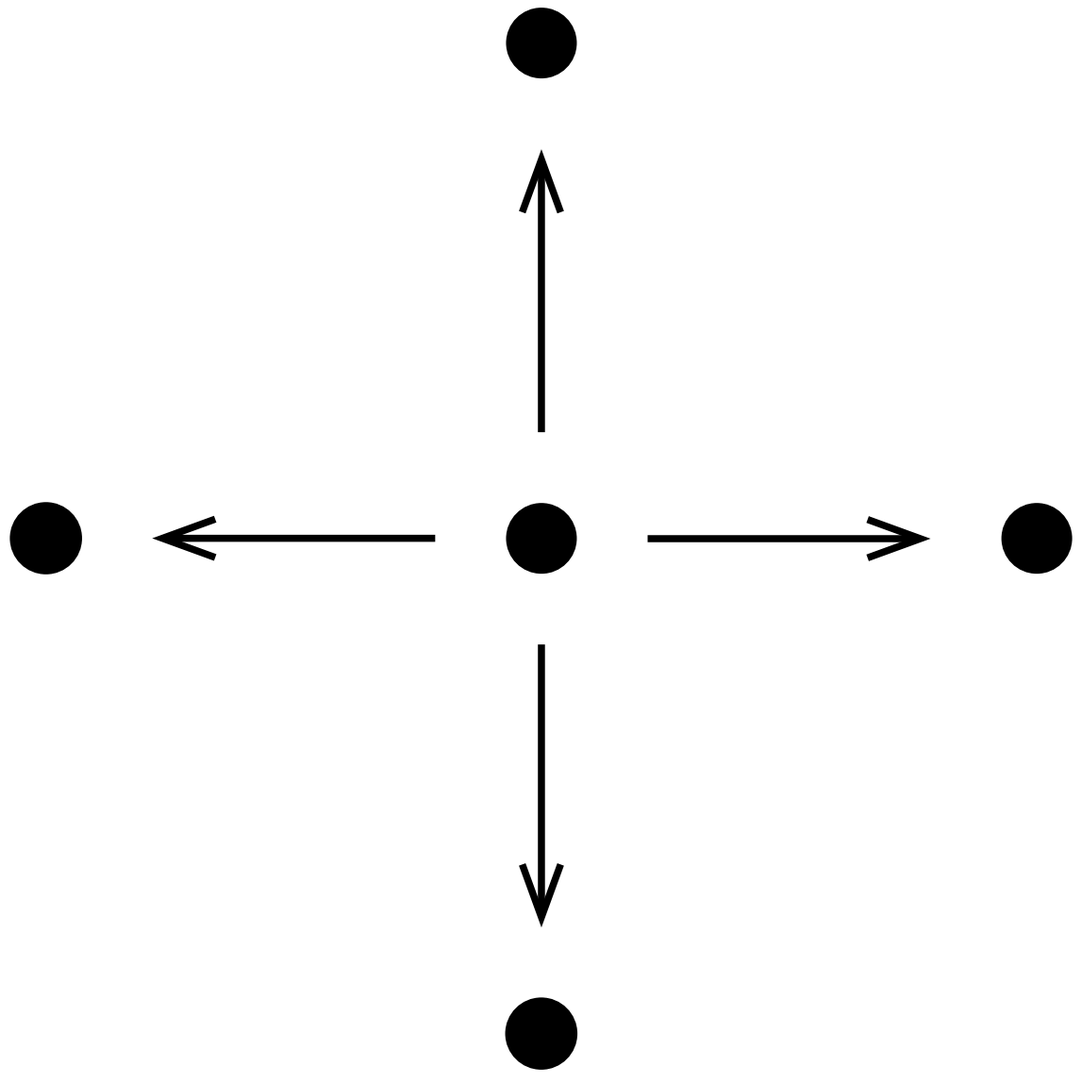
Local Adaptivity



Step Function



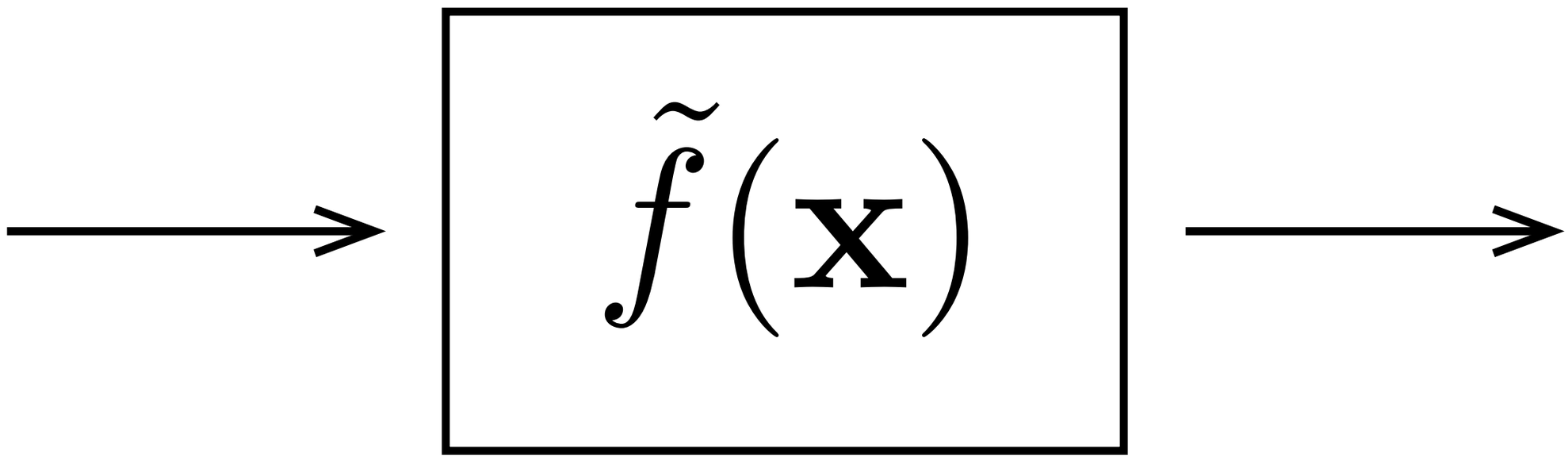
Which Direction to Go?



Hybrid Adaptivity

- Local refinements with respect to important dimensions

Conclusion



Conclusion

$$\tilde{f}(\mathbf{x}) = \sum_{\mathbf{k} \in \mathcal{K}} \Delta f(\mathbf{x_k}) e_{\mathbf{k}}(\mathbf{x})$$

Thank you!

Questions?